A Sufficient Statistics Approach to Measuring Forward-Looking Welfare

Baqaee, Burstein, and Koike-Mori

Discussion by Sewon Hur (FRB Dallas)

Texas Monetary Conference (SMU)

May 2024

Introduction

- Really nice paper!
- Beautiful example of using theory to extract useful information (welfare) from individual/household-level data

Today's discussion

- Ariel presented results from a simple model for intuition and then presented the general environment
- Let me walk through an even simpler example
- Some questions and potential applications

Super simple example

- 2-period lived agents
- Rentiers finance consumption with initial wealth w
- Non-rentiers additionally have income y and face borrowing constraints
- CRRA utility

Rentier's problem

- Problem of rentier with initial wealth w can be written as

$$V_t^0(w) = \max_{c_t^0, c_{t+1}^1, a_{t+1}^1} u\left(c_t^0\right) + \beta u\left(c_{t+1}^1\right)$$
$$\text{s.t. } p_t c_t^0 + a_{t+1}^1 = w$$
$$p_{t+1} c_{t+1}^1 = R_{t+1} a_{t+1}^1$$

- With CRRA utility,

$$\boxed{c_{t+1}^1 = \left(\beta R_{t+1} \frac{p_t}{p_{t+1}}\right)^{1/\sigma} c_t^0}$$

Rentier's solution

- Substitute in to budget constraint,

$$
\begin{aligned}
& p_t c_t^0 + \frac{p_{t+1} c_{t+1}^1}{R_{t+1}} = w \\
\Rightarrow & p_t c_t^0 + \frac{p_{t+1}}{R_{t+1}} \left(\beta R_{t+1} \frac{p_t}{p_{t+1}} \right)^{1/\sigma} c_t^0 = w \\
\Rightarrow & p_t c_t^0 = \underbrace{\left[1 + \beta^{1/\sigma} \left(R_{t+1} \frac{p_t}{p_{t+1}} \right)^{\frac{1-\sigma}{\sigma}} \right]^{-1}}_{b_t^0} w
\end{aligned}
$$

- Note that $\frac{b_{t+1}^1}{b_t^0} = \frac{1 - b_t^0}{b_t^0} = \beta^{1/\sigma} \left(R_{t+1} \frac{p_t}{p_{t+1}} \right)^{\frac{1-\sigma}{\sigma}}$

Money Metric

- The money metric welfare of household in period t with wealth w in terms of t_0 dollars is given by

$$u\left(c_t^0(w)\right)+\beta u\left(c_{t+1}^1(w)\right)=u\left(c_{t_0}^0(\hat{u})\right)+\beta u\left(c_{t_0+1}^1(\hat{u})\right)$$

$$\Rightarrow \frac{1}{1-\sigma}\left(b_t^0\frac{w}{p_t}\right)^{1-\sigma}+\beta\frac{1}{1-\sigma}\left(b_{t+1}^1\frac{R_{t+1}w}{p_{t+1}}\right)^{1-\sigma}$$

$$=\frac{1}{1-\sigma}\left(b_{t_0}^0\frac{\hat{u}}{p_{t_0}}\right)^{1-\sigma}+\beta\frac{1}{1-\sigma}\left(b_{t_0+1}^1\frac{R_{t_0+1}\hat{u}}{p_{t_0+1}}\right)^{1-\sigma}$$

$$\Rightarrow \left(w\frac{b_t^0}{p_t}\right)^{1-\sigma}\left[1+\beta\left(\frac{b_{t+1}^1}{b_t^0}\frac{R_{t+1}p_t}{p_{t+1}}\right)^{1-\sigma}\right]$$

$$=\left(\hat{u}\frac{b_{t_0}^0}{p_{t_0}}\right)^{1-\sigma}\left[1+\beta\left(\frac{b_{t_0+1}^1}{b_{t_0}^0}\frac{R_{t_0+1}p_{t_0}}{p_{t_0+1}}\right)^{1-\sigma}\right]$$

Money Metric

- Substitute $\frac{b_{t+1}^1}{b_t^0} = \beta^{1/\sigma}\left(R_{t+1}\frac{p_t}{p_{t+1}}\right)^{\frac{1-\sigma}{\sigma}}$ to obtain:

$$\left(w\frac{b_t^0}{p_t}\right)^{1-\sigma}\left[1+\beta^{1/\sigma}\left(R_{t+1}\frac{p_t}{p_{t+1}}\right)^{\frac{1-\sigma}{\sigma}}\right]$$

$$=\left(\hat{u}\frac{b_{t_0}^0}{p_{t_0}}\right)^{1-\sigma}\underbrace{\left[1+\beta^{1/\sigma}\left(R_{t_0+1}\frac{p_{t_0}}{p_{t_0+1}}\right)^{\frac{1-\sigma}{\sigma}}\right]}_{b_{t_0}^0}$$

- Taking logs and reorganize:

$$\boxed{\log(\hat{u}) = \log(w) - \log\left(\frac{p_t}{p_{t_0}}\right) + \frac{\sigma}{\sigma-1}\log\left(\frac{b_t^0}{b_{t_0}^0}\right)}$$

Special version of Proposition 1

- The money metric welfare of household in period t with wealth w in terms of t_0 dollars is given by

$$\log(\hat{u}) = \log(w) - \log\left(\frac{p_t}{p_{t_0}}\right) : \text{ adjusts for price level differences}$$
$$+ \frac{1}{1 - EIS} \log\left(\frac{b_t^0}{b_{t_0}^0}\right) : \text{ adjusts for intertemporal prices}$$

- If $EIS = \frac{1}{\sigma} > 1$, $b_t^0 < b_{t_0}^0$ reflects better savings opportunities, requiring compensation

Non-rentier's problem

- Problem of non-rentier with initial wealth w and income y

$$V_t^0(w, y) = \max_{c_t^0, c_{t+1}^1, a_{t+1}^1} u\left(c_t^0\right) + \beta u\left(c_{t+1}^1\right)$$

$$\text{s.t. } p_t c_t^0 + a_{t+1}^1 = w$$

$$p_{t+1} c_{t+1}^1 = y + R_{t+1} a_{t+1}^1$$

$$a_{t+1}^1 \geq -\bar{a}$$

- If borrowing constraint is non-binding,

$$b_{n,t}^0 = b_{r,t}^0$$

$$\hat{u} = w + \frac{y}{R_{t+1}}$$

Money metric for non-rentier

- If borrowing constraint is binding,

$$b^0_{n,t} = \frac{w + \bar{a}}{w + \frac{y}{R_{t+1}}} < b^0_{r,t}$$

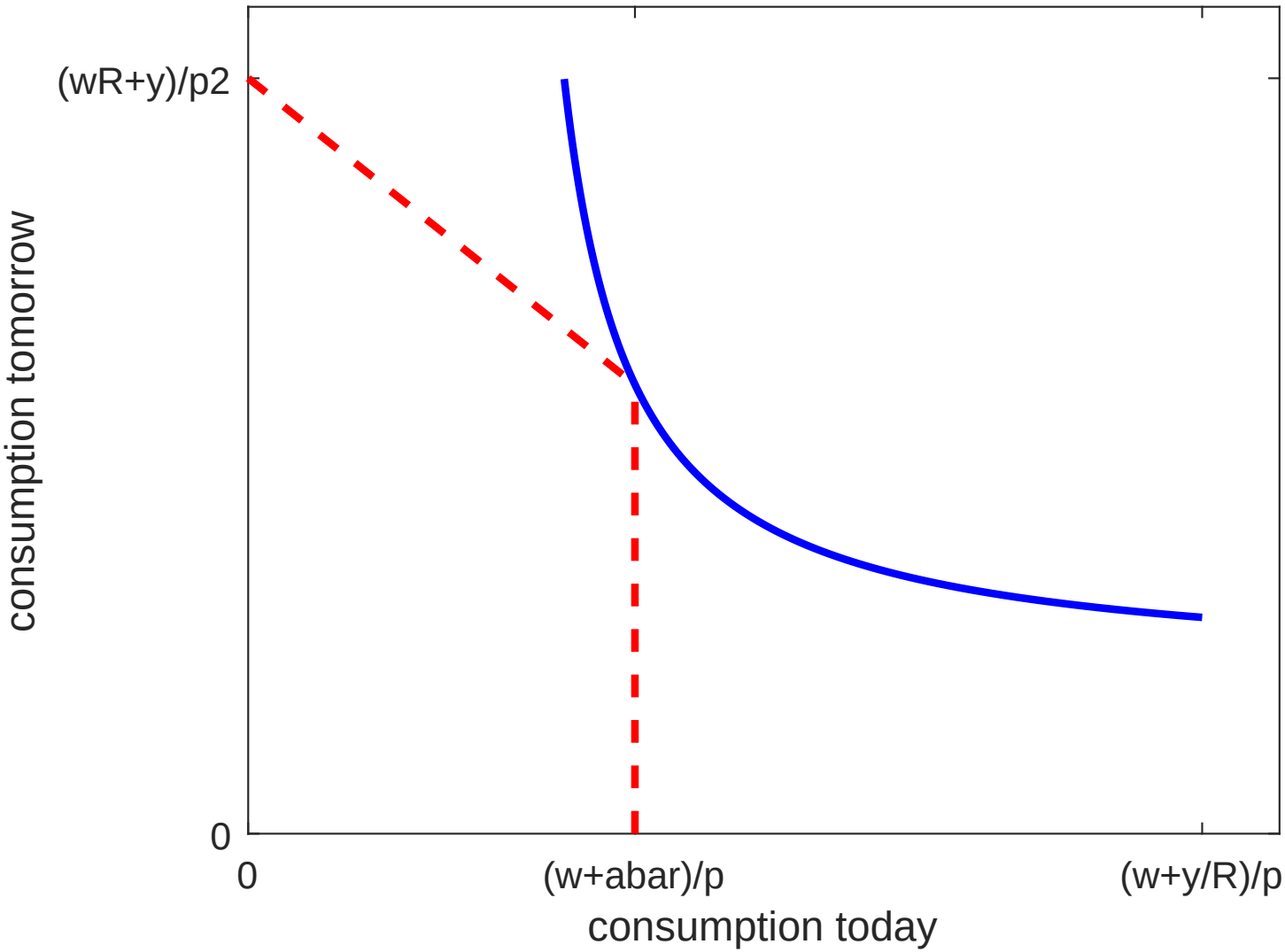
$$\hat{u} < w + \frac{y}{R_{t+1}}$$

- It is straightforward to show that

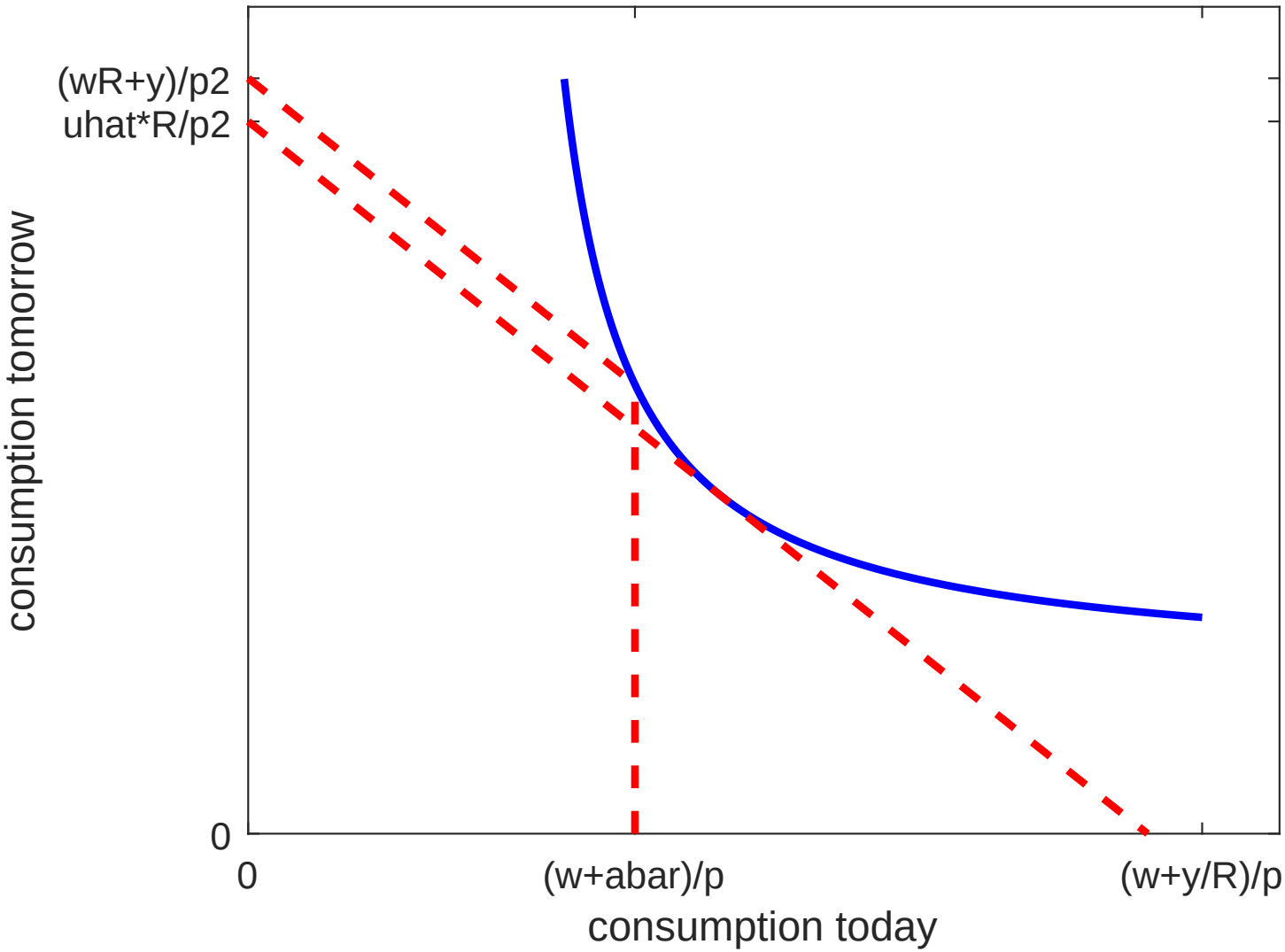
$$\frac{\partial \hat{u}}{\partial \bar{a}} \geq 0$$

meaning that relaxing borrowing constraints improve the money-metric weflare

Kinked budget constraint



Money metric welfare non-rentier

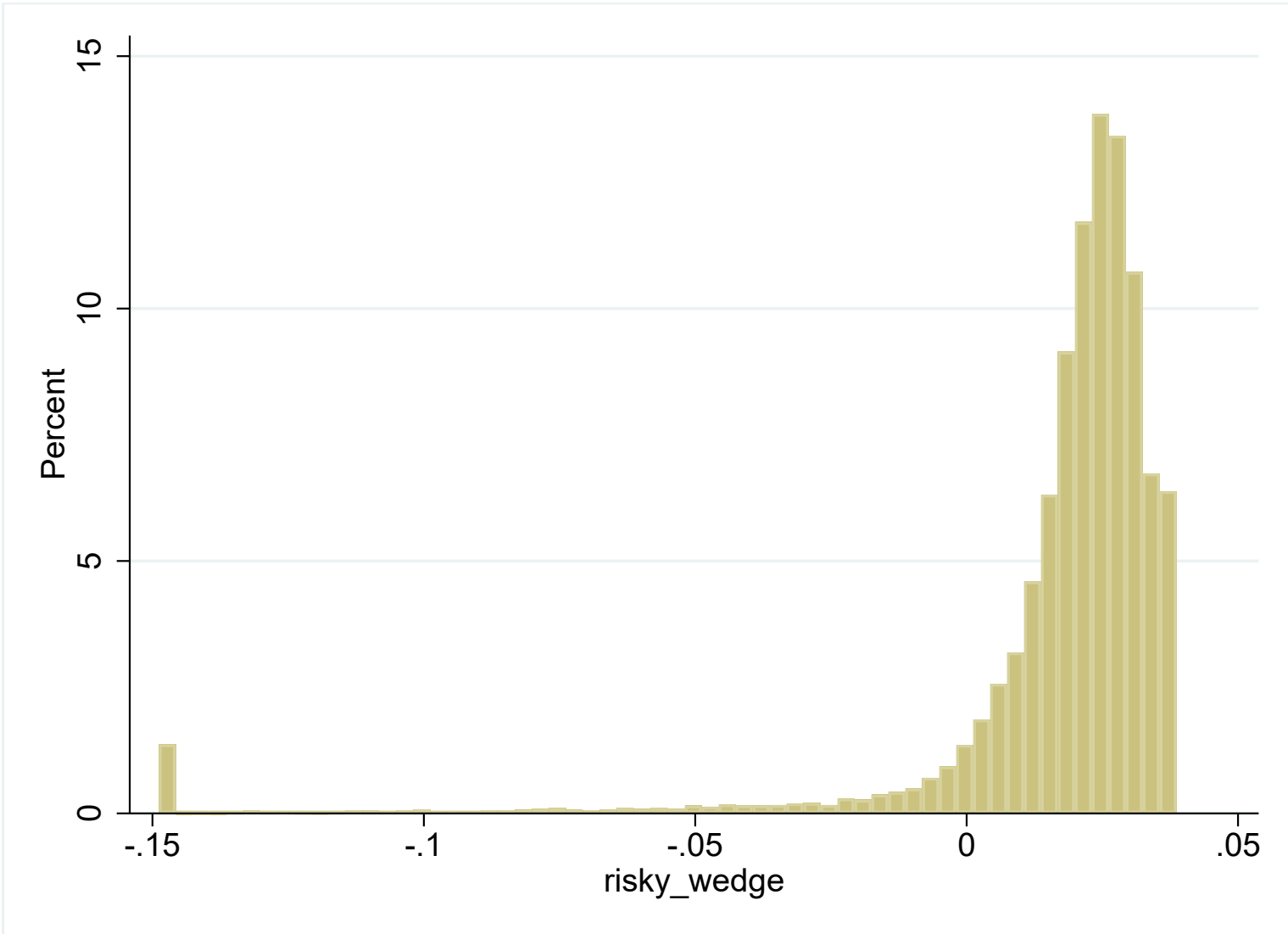


A ~~question~~, an observation rather

- I have an old paper (dormant until recently) with Chris Telmer, where we similarly use theory to extract information from household consumption/savings data.
- Similar idea/approach, but very different set of assumptions/implications
- We use a benchmark life-cycle portfolio-choice model to solve for wedges (à la Chari/Kehoe/McGrattan; Hsieh/Klenow) to rationalize household consumption/savings/risky allocations (PSID)
- Heteroegeneous wedges can reflect heterogeneous preferences for discounting/risk, borrowing constraints, heterogeneous returns, etc.

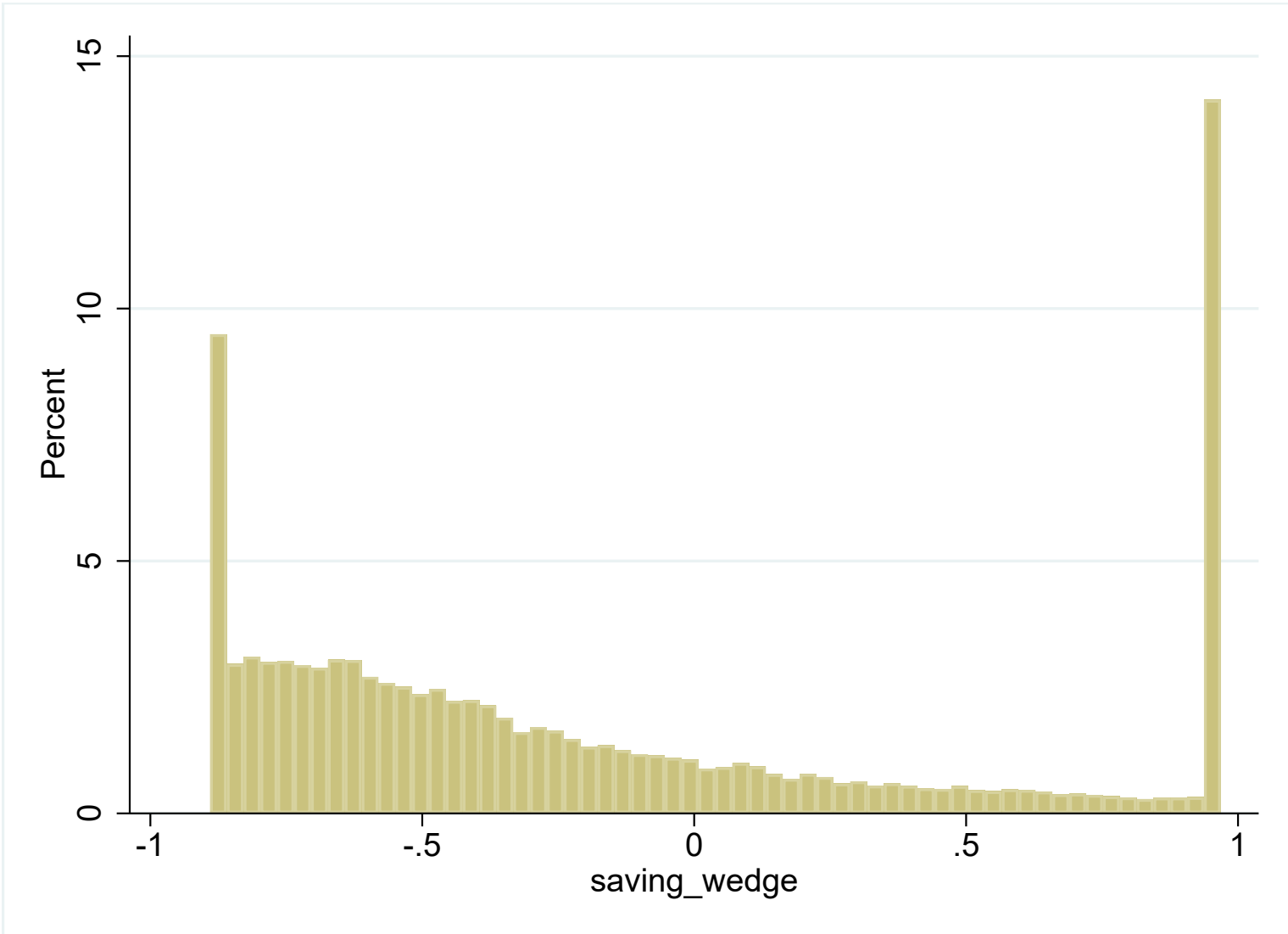
Risky wedge

- Risky wedge is very heterogenous across households



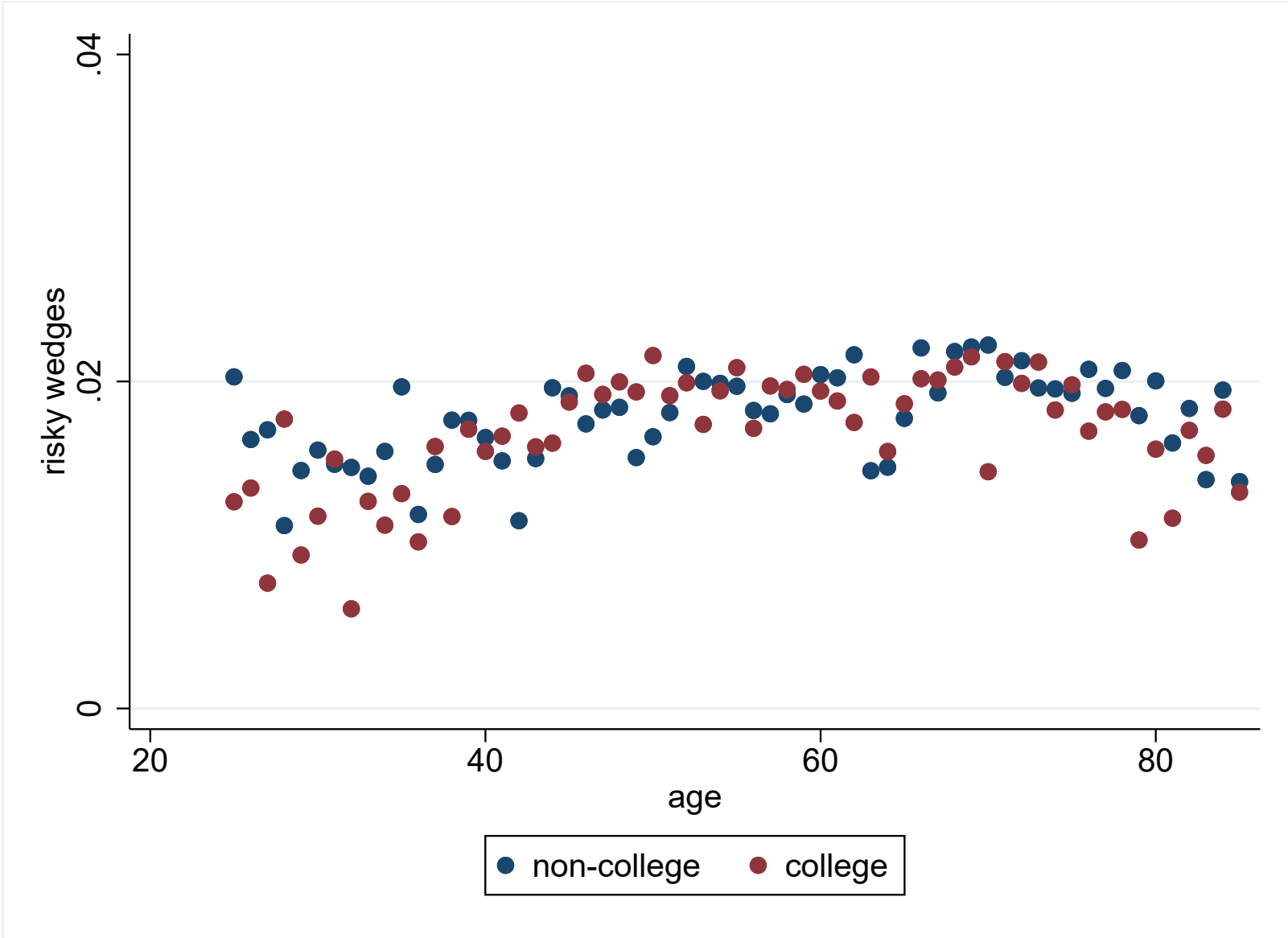
Saving wedge

- Savings wedge is also very heterogenous across households



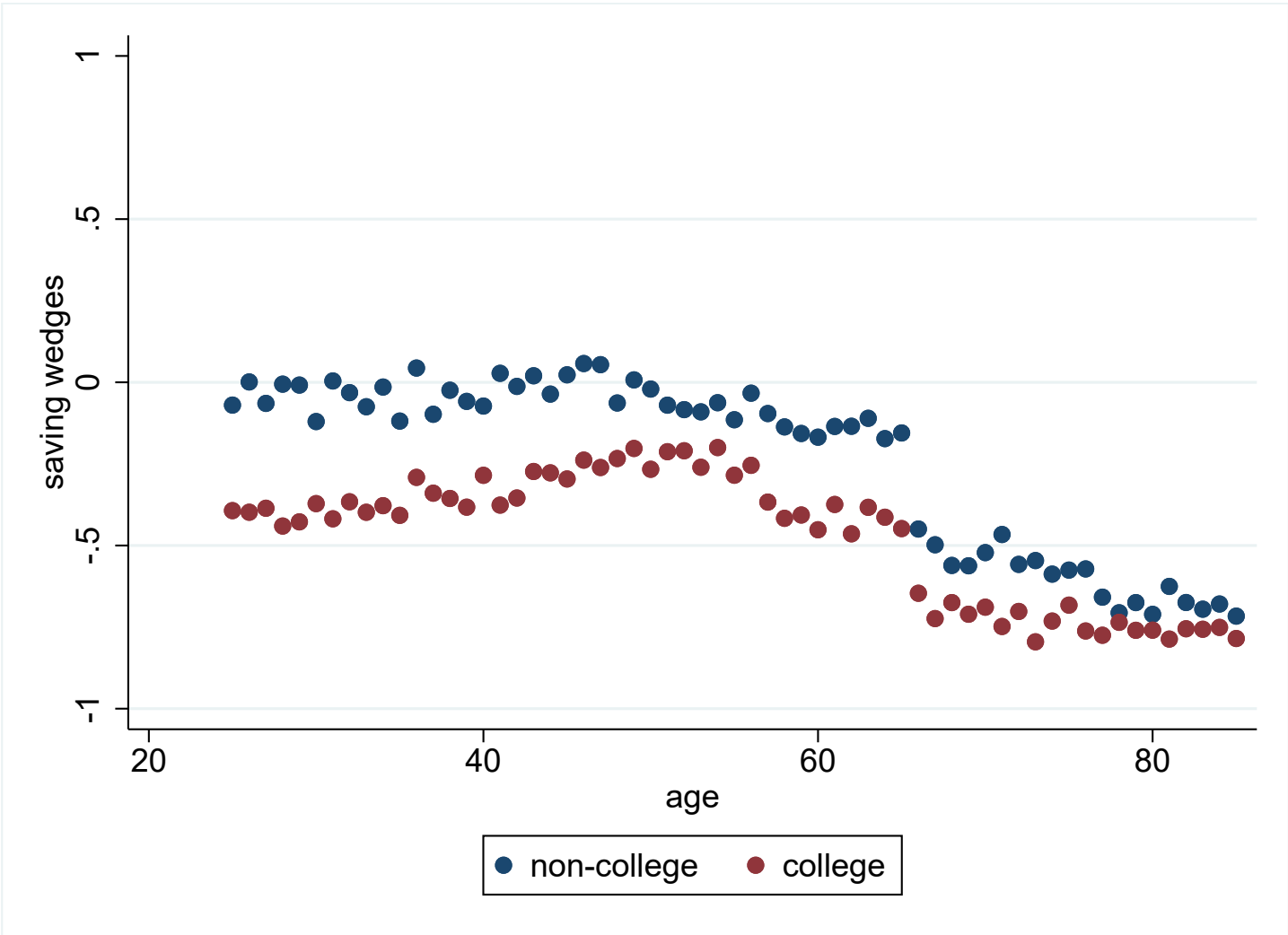
Risky wedge and education

- Risky wedge does not depend too much on age and education



Saving wedge and education

- Savings wedge not too dependent on working age vs. retirees
- Education seems to be an important determinant



Main takeaway

- From the lens of our analysis, savings wedge is quite heterogeneous
- This could reflect different savings opportunities, different discount factors, etc.
- How does this affect our interpretation of the non-rentier welfare results?

Wishlist and final thoughts

- An illustrative example that provides intuition for the non-rentier results would be very helpful. Perhaps a simple two-period model with non-homothetic preferences.
- How does welfare from job loss depend on education, income, and wealth etc., in addition to age.
- Job loss during aggregate downturn versus idiosyncratic job loss
- Measure the impact of trade (connect to the China trade shock literature)
- Measure the distribution of welfare of the Great Recession?
- Great thought-provoking paper, learned a lot!

Thanks everyone for sticking around, and especially to our amazing hosts!