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Unequal Climate Policy in an Unequal World[1]

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Abstract

We study climate policy in an economy with heterogeneous households, two types of goods (clean and dirty), and a climate externality from the dirty good. Using household expenditure and emissions data, we document that low-income households have higher emissions per dollar spent than high-income households, making a flat carbon tax regressive. We build a model that captures this fact and study climate policies that are neutral with respect to the income distribution. We show that the constrained optimal carbon tax in a heterogeneous economy is heterogeneous: Higher-income households face a higher rate. If uniformity of the carbon tax is desired, this property must be imposed as an additional constraint. In this case, the tax is lower than the unconstrained carbon tax. Finally, we embed this model into a standard incomplete markets framework to quantify the policy effects on the economy, climate, and welfare, and we find a Pareto-improving result. The uniform climate policy is welfare-improving for every household.

KEYWORDS: carbon tax, inequality, consumption, welfare, climate change.

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1 Introduction

Climate change is an increasingly important issue policymakers must address. While economists widely support carbon taxes as an effective tool to reduce emissions, concerns persist about their disproportionate impact on low-income households, whose consumption baskets tend to be more carbon-intensive. In principle, concerns regarding inequality could be addressed through other taxes and transfers, enabling the carbon tax to focus solely on internalizing the climate externality. However, the availability of these instruments may often be limited. In this paper, we study optimal climate policy in an unequal world and how the tools available shape the design of the carbon tax.

We build a simple model with clean and dirty goods and households that are heterogeneous by income. Consumption of dirty goods adds carbon to the atmosphere, generating a negative climate externality. When the social planner is allowed to transfer resources across households, the optimal carbon tax is uniform. Furthermore, the carbon tax is identical to that from a representative agent economy when the social welfare function is utilitarian. When the planner cannot make transfers between households, the constrained-efficient carbon tax is heterogeneous. If uniformity of the carbon tax is desired, this property must be imposed as an additional constraint. In this case, we show when and how the tax deviates from the optimal carbon tax formula. Importantly, in quantitative exercises we find that this tax policy leads to a Pareto improvement.

In the first part of the paper, we combine detailed household expenditure and emissions data to document that the embodied carbon content of household expenditures decreases in income and in wealth. These facts are robust to controlling for household age, education, and family size. These differences can mainly be attributed to low-income households' greater expenditure share on utilities, transportation, and groceries. Motivated by these findings, we incorporate non-homothetic preferences into our model and discipline its calibration by targeting the differences of emission intensities in consumption across households.

To study optimal climate policies in a world with inequality, we build an endowment economy with permanent inequality, where households differ in their labor endowments and supply labor inelastically. Clean and dirty consumption goods are produced using labor. Because the only dynamic variable is the stock of carbon, the household problem is effectively static. This lends tractability to the model and allows us to characterize policies in closed-form. When the planner is not restricted in its instruments, the optimal climate policy is a uniform carbon tax that is equal to the social cost of carbon. This cost is the price of the externality and equals the marginal social damage of carbon divided by the social value of consumption. This means that the planner uses the social value of consumption—the Pareto-weighted average of marginal utilities of consumption—to price the climate externality. In the benchmark case, a utilitarian planner can eliminate inequality by transferring resources across households. This produces the same social value of consumption as that from a representative agent economy, and as a result, the optimal carbon tax is the same in the two economies.

Because our objective is to study optimal climate policies in the presence of inequality and not as a means to address inequality, we focus on cases in which initial inequality is preserved. One way to achieve this is by considering Negishi Pareto weights, where high-income households receive relatively more weight in social welfare. Because these households have lower marginal utilities of consumption, the social value of consumption is lower, resulting in a more aggressive climate mitigation under Negishi. Our central findings come from an alternative approach, where we adhere to a utilitarian planner but restrict its ability to transfer resources across households. We do this by imposing constrained efficiency as in Davila et al. (2012) under which each household's carbon tax payment is rebated back to it lumpsum. As a result, the constrained-efficient carbon tax is heterogeneous. To price the climate externality, the constrained planner uses each household's private value of consumption (i.e., its marginal utility), instead of the social value of consumption. Because the marginal utility of consumption is decreasing with income, the constrained-efficient carbon tax is increasing in income. That is, the constrained-efficient carbon tax is progressive.

Next, we study the optimality of a uniform carbon tax, the policy most commonly proposed in public debates. Since a uniform carbon tax is not the solution to an optimal policy design problem in which the planner is constrained from transferring resources across households, we must impose uniformity of the tax rate as an additional constraint. Even though this planner cannot do away with inequality, the uniform constrained-efficient carbon tax nevertheless follows the same formula as that of the unconstrained planner. That is, the uniform constrained-efficient carbon tax is set to the marginal social damage of carbon divided by the social value of consumption. However, because marginal utilities are heterogeneous in this case, the social value of consumption can differ from that of the unconstrained planner.

When the marginal utility function is sufficiently convex, climate mitigation under a uniform constrained planner is moderated relative to the utilitarian optimal benchmark. The intuition for this result is simple. Because the weighted average of marginal utilities is greater than the marginal utility of average consumption, the social value of consumption is higher for the constrained planner. Unable to eliminate initial inequality and prohibited from assigning heterogeneous tax rates, the planner accounts for the high marginal utilities of low-income households by effectively putting more weight on the social value of consumption relative to the social damage from carbon emissions.

With further functional form assumptions on preferences, we can provide an alternative implementation of the uniform constrained-efficient allocation that does not include individual rebates. This may be more practical because it reduces the informational burden on policymakers. Specifically, we show that the uniform constrained-efficient allocation can be decentralized through a combination of three instruments: a uniform carbon tax, a uniform clean subsidy, and a uniform lump-sum transfer.

We assess the welfare and climate implications of these carbon taxes through a numerical exercise. Among climate policies that maintain initial inequality, we find that the heterogeneous constrained-efficient carbon tax achieves the highest average welfare gain and the most significant long-term temperature reduction. However, due to its progressive structure, the constrained-efficient tax faces limited support among high-income households, which bear a greater tax burden and experience welfare losses. In contrast, the uniform constrained-efficient tax secures full support (i.e., it is Pareto-improving) and achieves the highest average welfare gain from among the uniform taxes. Nevertheless, universal support comes at the expense of the climate. The long-term global temperature reduction is the smallest under this tax policy. Greater temperature reduction occurs under the uniform Negishi tax, but this policy faces substantial opposition, now from the bottom of the distribution.

Finally, we embed the simple model in a standard incomplete markets model with idiosyncratic labor income risk. This empirically grounded economy captures existing inequality and fiscal policy, enabling us to quantify both the effectiveness and the distributional consequences of carbon taxes. The economic parameters are calibrated to match key features of the US income and wealth distributions as well as US fiscal policy. Climate parameters are calibrated to ensure that economic activity generates global emissions consistent with the data and contributes to temperature rises that are consistent with recent estimates.

We solve the model under several climate policy scenarios. First, we use the uniform constrained-efficient tax formulas to solve for the uniform carbon tax combined with a household-specific transfer that fully rebates each household's carbon tax payment. The equilibrium carbon tax schedule begins at $41 per ton and rises gradually over time to a long-run value of $78 per ton, reflecting the increasing social cost of carbon. Next, we solve for a proxy to the heterogeneous constrained-efficient carbon tax where each household's carbon tax rate is a function of its current labor productivity. The value of the initial carbon tax ranges from \$10 per ton for the poorest households to \$8,400 per ton for the richest. Just as in the uniform case, all tax rates rise over time as the social cost of carbon increases.

Both policies reduce carbon emissions substantially and lower temperatures relative to business-as-usual (BAU), but the greatest climate gains emerge far in the future. The heterogeneous carbon tax, which induces the strongest reduction in carbon accumulation, lowers long-run temperatures by approximately 1.5 degrees Celsius relative to BAU. These carbon tax policies are welfare improving for the majority of households. The uniform carbon tax, in particular, achieves a Pareto improvement relative to BAU. Welfare gains in the initial period of transition are small on average (about 0.05 percent in consumption equivalence units); however, they grow substantially over time due to accumulated climate improvements relative to BAU.

Comparing the uniform and heterogeneous tax cases, the latter produces a somewhat lower temperature path and a slighter higher average welfare gain. In both cases, the welfare benefit is greatest among the wealthy. Because these households have much lower marginal utilities of consumption, they place greater value on climate improvements than the poor do. With a uniform carbon tax, welfare gains increase monotonically in both wealth and in income (i.e., labor productivity). In contrast, by taxing high-income households more, the heterogeneous carbon tax leads to more equitable welfare gains across income conditional on having the same level of wealth.

We show that our main quantitative result—that a uniform carbon tax with rebates leads to a Pareto improvement—is robust to alternative parameter and modeling assumptions. In particular, when risk aversion is higher than in our baseline calibration, marginal utilities are even more convex, resulting in a higher social value of consumption, a lower carbon tax, and a more moderate climate mitigation. Making households more impatient than in our baseline leads to a lower social cost of carbon and a lower tax rate, since severe climate damages appear in the future and are more heavily discounted. In both cases, however, the uniform carbon tax and rebate policy lead to a welfare gain for all. Finally, we extend the model to allow for heterogeneous climate damages, permitting damages to be larger for low-income households. We show that for plausible variation in the elasticity of climate damages to income, uniform carbon taxes with rebates lead to a Pareto improvement.

Literature Review. This paper contributes to a growing body of literature that builds on the foundational work of Nordhaus and Boyer (2003) (and its contemporary version, Golosov et al. 2014) on representative agent neoclassical growth economies with climate dynamics by incorporating heterogeneous agent economies with incomplete markets. Recent notable contributions come from Krusell and Smith Jr. (2022), Hillebrand and Hillebrand (2019), Fried et al. (2018), Douenne et al. (2023), Belfiori and Macera (2024), and Fried et al. (2023), and within a spatial economic framework from Cruz and Rossi-Hansberg (2023) and Conte et al. (2022).

Our paper is most closely related to a recent strand of the literature that considers fiscal reforms to address climate change in economies with heterogeneous agents. Fried et al. (2023) build a life-cycle model where households have non-homothetic preferences for dirty consumption and study the welfare consequences of alternative ways to rebate revenue from a given carbon tax. In contrast, we theoretically characterize the constrained-efficient climate policy (taxes and transfers) in the economy with heterogeneity and use these characterizations to inform the policies studied in our quantitative exercises. Jacobs and van der Ploeg (2019) and Douenne et al. (2023) also study optimal policy to resolve climate change and inequality concerns when carbon taxes are considered jointly with other distortionary fiscal instruments. We differ from all three of these papers in that we design carbon taxes and transfers to fix a climate externality in the presence of inequality rather than to address climate and inequality simultaneously.

In order to focus on climate policy as a means to fix the climate externality and not to address inequality, we impose constrained efficiency. That is, we restrict resource transfers across agents—as in the pioneering work by Davila et al. (2012). Belfiori and Macera (2024) and Bourany (2024) also study constrained-efficient allocations within climate-economy models with idiosyncratic risk and incomplete markets. This paper differs from them by focusing on household consumption inequality, as opposed to regional heterogeneity across countries. We share with these papers a careful consideration of the tension between redistributive motives and efficiency in the policy design, which naturally arises in a heterogeneous agent economy.

The paper also connects to other papers that study the distributional role of carbon tax revenue, such as Rausch et al. (2011), Pizer and Sexton (2019), Fullerton and Monti (2013), and Goulder et al. (2019). Our empirical work is related to Grainger and Kolstad (2010) and Sager (2019), who also document how emissions embodied in household expenditures vary with income. Consistent with our empirical finding, Känzig (2023) documents that carbon prices affect the consumption of low-income households relatively more. However, our paper differs in scope and methodology, as we set up an optimal policy design problem to examine carbon pricing policies. Also, while the primary mechanism for the decline in consumption in

Känzig (2023) is a reduction in income, the key driver in our model is the change in relative prices.

The remainder of this paper is organized as follows. Section 2 uses data on household expenditure and embodied emissions to document how emission intensities differ across income and wealth. Section 3 presents a simple model with unequal agents and climate change, and Section 4 provides the analytical characterizations. Section 5 presents the quantitative model, the calibration, and the quantitative results. Finally, Section 6 concludes.

2 Data

In this section, we document the embodied emissions content of household expenditures and how it differs across the income and wealth distribution. We combine data on household expenditures, income, and liquid wealth from the Consumer Expenditure Survey (CEX) with data on embodied emissions from the Environmental Protection Agency (EPA). The emissions data include carbon dioxide (CO_2) emissions and other greenhouse gases such as methane and nitrous oxide, converted to CO_2 equilvalents using the Intergovernmental Panel on Climate Change (IPCC) Assessment Report's 100-year global warming potential. It covers supply chain emissions (from cradle to factory gate) and margins (from factory gate to shelf, including transportation, wholesale, and retail).

To combine the expenditure data with the emissions data, we first construct a concordance to map 671 Universal Classification codes (UCC) of CEX expenditures to 394 North American Industry Classification System (NAICS) codes used in the emissions dataset (EPA). For example, the UCC code 100210 (cheese) is linked to the NAICS-6 code 311513 (cheese manufacturing), which is associated with 1.585 kilograms of CO_2-equivalent embodied emissions per 2018 dollar spent. As another example, the UCC code 560110 (physician services) corresponds to the NAICS-4 code 6211 (offices of physicians), which is associated with 0.082 kilograms of CO_2-equivalent embodied emissions per dollar.[2]

The CEX microdata consist of two surveys: The diary survey collects detailed information on a subset of household expenditures (especially for groceries, such as flour, rice, and white bread) for two consecutive weeks and the interview survey collects more aggregated expenditures that cover most household expenditures (e.g., food at home, college tuition, camping equipment, and airline fares) for one year. Though the two surveys are not linked, we use the detailed food and beverage expenditures from the diary survey to estimate an embodied

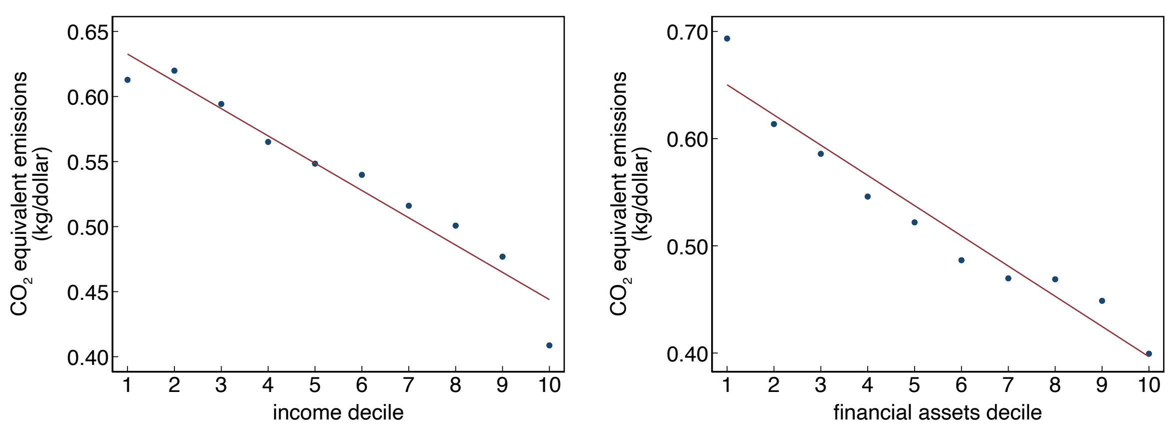
[2]The full concordance is provided online.

emission function and apply it to the interview data on food and beverages at home. For all other interview expenditure categories, we use the constructed UCC-NAICS concordance to directly calculate embodied emissions. Finally, we include direct tailpipe emissions, which are associated with about 9 kilograms of CO_2 per gallon driven (EPA).[3]

Using the constructed dataset, we document that the emission intensity of household expenditures decreases with income and with wealth. That is, the expenditures of lower-income and lower-wealth households are associated with higher embodied emissions per dollar spent. Figure 1 plots the average embodied emissions per dollar spent by income and liquid wealth decile.[4] Emission intensity is clearly decreasing in both income and wealth: Compared with the highest income and wealth households, the expenditure of the lowest-income and lowest-wealth households is associated with about 25 additional kilograms of CO_2-equivalent emissions per $100 spent.

Figure 1: Embodied emissions

(a) Income (b) Wealth



We further break down the source of this variation by broad expenditure categories. In Table 1, we can see that low-income households' expenditure baskets are more tilted toward expenditure categories with the highest emission intensities (utilities, transportation, and food and beverages at home), relative to high-income households. High-income households spend relatively more on all other expenditures, which are associated with lower emission intensities (including entertainment, education and child care, and health care).

[3]See https://www.fueleconomy.gov/feg/label/calculations-information.shtml.

[4]The CEX contains data on liquid wealth, including the value of checking, savings, money market accounts, and certificates of deposit. In Appendix A, we show that the results are robust to using the Panel Study of Income Dynamics, which contains a more complete representation of household wealth.

Table 1: Embodied emissions and expenditure shares

Expenditure category	Embodied emissions (CO_2 kg/dollar)	Expenditure shares (percent)	
		Low income	High income
Utilities	1.71	9.1	5.1
Transportation	1.16	18.4	16.0
Food and beverages at home	0.80	14.1	7.7
Other expenditures	0.11	58.4	71.2

High and low income correspond to the top and bottom deciles of income, respectively, conditional on working age.

To document the relationship between income and wealth and embodied emissions intensities more systematically, we regress the intensities on the natural logs of income and wealth in Table 2. Columns (1)–(2) demonstrate that wealth and income are negatively associated with embodied emission intensities, statistically significant at the 1 percent level. Column (3) shows that this result is robust to controlling for education, age, and family size fixed effects. These effects are also economically significant: Using the coefficients in column (3), increases of one standard deviation in log income and wealth are associated with increases of 2.1 and 6.2 percentage points in the embodied emission intensities.

Table 2: Embodied emission intensity

	(1)	(2)	(3)
Wealth	-2.48^{***}		-1.87^{***}
	(0.131)		(0.175)
Income		-4.70^{***}	-2.14^{***}
		(0.243)	(0.612)
Observations	1488	5102	1488
Adjusted R^2	0.195	0.068	0.241

Standard errors in parentheses. (3) additionally includes college, age, and family size fixed effects. *** represents statistical significance at the 1 percent level.

3 A Simple Model

In the previous section, we documented that the emissions embodied in household expenditures substantially vary with income and wealth, suggesting that a carbon tax can have unequal consequences across households. In this section, we develop a simple model of unequal households and climate change to study how the optimal carbon tax depends on underlying inequality.

Consider an economy populated by a continuum of households, indexed by i with measure μ_i. There are two consumption goods, clean and dirty: c_{ct} and c_{dt}. Consumption of the dirty good adds carbon to the atmosphere, S_t, which evolves according to:

$$S_{t+1} = (1-\delta)S_t + \upsilon \sum_i \mu_i c^i_{dt} \tag{1}$$

where δ is the natural rate of carbon re-absorption and υ is the carbon content of dirty good consumption.

Households' preferences over consumption are given by:

$$\sum_{t=0}^{\infty} \beta^t \left[u(c_{ct}, c_{dt}) - x(S_{t+1})\right] \tag{2}$$

where $x(S)$ is the climate damage function with $x'(S) > 0$ and $x''(S) > 0$. The function x subsumes the welfare losses from the presence of carbon in the atmosphere, and we assume these losses take the form of a utility cost. In the quantitative exercise, we additionally consider the mapping from the carbon stock to the global temperature, and we include in x all climate-related welfare losses regardless of whether they are utility or output related. Therefore, x in the model represents global climate change.

Households are endowed with ε_i units of labor (inelastically supplied) and choose consumption to maximize utility (2) subject to the following set of budget constraints,

$$p_t(1+\tau^i_t)c^i_{dt} + c^i_{ct} \leq w_t\varepsilon^i + T^i_t, \tag{3}$$

for every period t, where p_t is the relative price of dirty to clean consumption and w_t is the wage. Additionally, τ^i_t is a carbon tax on dirty good consumption and T^i_t is a lump-sum tax or transfer.

A government collects carbon taxes and uses the proceeds to finance government spending and lump-sum rebates/taxes to households. There are no other distortionary taxes available. The budget constraint of the government for every period t is

$$\sum_i p_t\tau^i_t\mu_i c^i_{dt} = G_t + \sum_i \mu_i T^i_t. \tag{4}$$

There are two production units, the clean and the dirty good producers indexed by j. A representative firm uses labor as the only input in each sector according to a linear technology. Thus, the aggregate production of the clean and the dirty good is given by $Y_{ct} = N_{ct}$ and $Y_{dt} = N_{dt}$, respectively.

Finally, market clearing for each period t requires that

$$N_{ct} + N_{dt} = \sum_i \mu_i \varepsilon^i \tag{5}$$

$$N_{ct} = \sum_i \mu_i c^i_{ct} + G_t \tag{6}$$

$$N_{dt} = \sum_i \mu_i c^i_{dt} \tag{7}$$

Definition 1 (Competitive Equilibrium with Carbon Taxes) *A competitive equilibrium with taxes* $\{\tau^i_t, T^i_t\}_{t=0}^{\infty}$ *is a sequence of prices* $\{p_t, w_t\}_{t=0}^{\infty}$ *and allocations* $\left\{\{c^i_{jt}, N_{jt}\}_{j=c,d}\right\}_{t=0}^{\infty}$ *such that* (*i*) *given prices and taxes, households choose* $\{c^i_{ct}, c^i_{dt}\}_{t=0}^{\infty}$ *to maximize (2) subject to (3) for all i;* (*ii*) *given prices, firms of sector* $j = \{c, d\}$ *choose* $\{N_{jt}\}_{t=0}^{\infty}$ *to maximize profits;* (*iii*) *the government budget constraint* (4) *is satisfied;* (*iv*) *the stock of atmospheric carbon evolves according to* (1)*, and* (*v*) *prices clear the markets.*

At an interior solution, household and firm optimality conditions imply:

$$\frac{u^i_{dt}}{u^i_{ct}} = 1 + \tau^i_t \tag{8}$$

which states that the marginal rate of substitution between clean and dirty consumption equals the relative price for every period t. In this economy, profit maximization on the firm's side implies that $p_t = w_t$ in every period t and we normalize this price to one.

4 Analytical Results

Our goal is to study the optimal carbon tax in an economy with inequality. However, the solution to this question depends on the tools available to the planner to deal with inequality. In this section, we start with an unrestricted planner (with arbitrary Pareto weights) that designs carbon taxes and individual transfers to address both inequality and carbon emissions. The general solution to this problem takes the form of a uniform carbon tax rate with heterogeneous transfers. The tax rate equals the social cost of carbon, which is defined as the ratio of the *marginal social damage* from a unit of carbon to the *social value of consumption*, a weighted average of households' marginal utilities of consumption. Throughout the section, the planner's problem will be altered, either by specifying an alternative social welfare function or, primarily, by restricting the planner's ability to address inequality. The differences in optimal policy across the various cases arise from how each planner measures the social value of consumption. We conclude the section with a simple numerical illustration, calibrated so that it is comparable to our richer quantitative model in Section 5, that measures the effects of policy on climate and on households' welfare.

4.1 Optimal Carbon Tax

Consider a government with access to a complete set of instruments, including individual-specific taxes and lump-sum transfers, and no financing needs. The government collects carbon taxes and all proceeds from taxation are rebated lump-sum. The optimal carbon-tax-and-transfer scheme implements the socially optimal allocation as a competitive equilibrium.

Definition 2 (Optimal Allocation) *Let $\{\alpha_i\}_{\forall i}$ be an arbitrary set of Pareto weights with $\sum_i \alpha_i = 1$. The socially optimal allocation is the sequence $\{c^{i\star}_{dt}(\alpha_i), c^{i\star}_{ct}(\alpha_i), S^\star_t\}^\infty_{t=0}$ that solves the social planner's problem, which is to maximize*

$$\sum_i \alpha_i \left[\sum_{t=0}^{\infty} \beta^t \left(u(c^i_{ct}, c^i_{dt}) - x(S_{t+1}) \right) \right] \tag{9}$$

subject to the carbon cycle (1) and the resource constraint,

$$\sum_i \mu_i c^i_{ct} + \sum_i \mu_i c^i_{dt} = \sum_i \mu_i \varepsilon^i. \tag{10}$$

The first order conditions for this problem are:

$$(c^i_{dt}) : \alpha_i u^i_{dt} - \upsilon \mu_i \sigma_t - \mu_i \lambda_t = 0, \tag{11}$$

$$(c^i_{ct}) : \alpha_i u^i_{ct} - \mu_i \lambda_t = 0, \tag{12}$$

$$(S_{t+1}) : -x'(S_{t+1}) + \sigma_t - \beta \sigma_{t+1}(1-\delta) = 0, \tag{13}$$

where $\beta^t \sigma_t$ and $\beta^t \lambda_t$ are the Lagrange multipliers on the carbon cycle and resource constraint, respectively. Iterating forward from (13), we have:

$$\sigma_t = \sum_{j=1}^{\infty} [\beta(1-\delta)]^{j-1} x'(S_{t+j}), \tag{14}$$

representing the present discounted sum of climate-induced damages associated with an additional unit of dirty consumption, which we will refer to as the *marginal social damage* of carbon.

Notice that equations (11)–(12) hold for all i. Thus, for all i

$$\lambda_t + \upsilon\sigma_t = \frac{\alpha_i}{\mu_i} u^i_{dt}, \tag{15}$$

$$\lambda_t = \frac{\alpha_i}{\mu_i} u^i_{ct}. \tag{16}$$

That is, weighted marginal utilities are equated across agents. This implies that, for all i, j,

$$\frac{u^i_{dt}}{u^i_{ct}} = \frac{u^j_{dt}}{u^j_{ct}}, \tag{17}$$

meaning that the marginal rates of substitution between goods are equated across agents.

Combine (15)–(16) to obtain:

$$1 + \frac{\upsilon\sigma_t}{\lambda_t} = \frac{u^i_{dt}}{u^i_{ct}}. \tag{18}$$

The optimality condition says that the marginal utility must be equal across goods, after taking into account the climate externality.

Uniform Carbon Taxes. It follows from a simple observation of optimality conditions (18) and (8) that the optimal Pigouvian tax that implements the socially optimal allocation is

$$\tau^\star_t \equiv \frac{\upsilon\sigma_t}{\sum_i \alpha_i u^i_{ct}}. \tag{19}$$

Given a choice of Pareto weights, lump-sum transfers are equal to $T^i_t(\alpha_i) = (1+\tau^\star_t)c^i_{dt} + c^i_{ct} - \varepsilon^i$, so that weighted marginal utilities are equated as in equations (15)–(16).

The carbon tax in (19) reflects the social cost of carbon, which is the ratio of the marginal social damage to the social value of consumption, measured in this case by the Pareto-weighted average marginal utility of clean consumption. Thus, a uniform carbon tax—consistent with the optimal policy in a representative agent framework—remains optimal in economies with heterogeneous agents, provided lump-sum transfers are available. The actual tax rate depends on the allocation, which in turn depends on the distribution of welfare weights. Importantly, the climate policy is not uniform in all aspects as it prescribes household-specific lump-sum transfers. Moreover, this uniform carbon tax potentially entails a significant redistribution of resources across households implemented through these transfers.

Utilitarian Optimal Carbon Tax. When the planner is utilitarian (i.e., $\alpha_i = \mu_i$), marginal utilities for both clean and dirty consumption are equalized across agents, meaning that the planner eliminates any inequality across agents. The optimal allocation coincides with the one that prevails in a representative agent economy, and the optimal tax is the same. Using (19), the utilitarian carbon tax, $\tau_t^{\mathbf{U}}$ is equal to

$$\tau_t^{\mathbf{U}} = \frac{\upsilon \sigma_t}{u_{ct}}. \tag{20}$$

where u_{ct} indicates the marginal utility of clean consumption. Because agents have equal consumption of both types, u_{ct} no longer depends on i and the social cost of carbon and the private cost of carbon are equal. In this case, a dichotomy appears regarding policy objectives and instruments: Transfers eliminate inequality within the economy, and the carbon tax addresses the climate externality. Simply put, if a planner is given a sufficient set of instruments, inequality does not matter for optimal carbon taxation.

Negishi Optimal Carbon Tax. The planner can, of course, resolve the existing inequality when unrestricted lump-sum transfers are available. This is true in general, regardless of the presence of a climate externality. Because our primary interest is in understanding how the presence of inequality affects optimal climate policy, rather than how climate policy can be used to reduce inequality, a natural exercise is to eliminate redistribution either by explicitly restricting the planner from redistributing resources or by ex ante reducing the planner's concern for it. The latter can be achieved by considering a planner with Negishi Pareto weights, since these weights are designed so that the optimal policy preserves initial inequality. The Negishi planner can undo inequality; it just does not want to. Negishi weights take the following form:

$$\alpha_i \equiv \frac{\frac{1}{u_\varepsilon^i}\mu_i}{\sum_j \frac{1}{u_\varepsilon^j}\mu_j}. \tag{21}$$

Here the welfare weights are proportional to the inverse of the marginal utilities of individual's *total* consumption, which we denote by u_ε^i to indicate that each household's total consumption is equal to their endowment. Using (19), the Negishi carbon tax, $\tau_t^{\mathbf{N}}$, is equal to

$$\tau_t^{\mathbf{N}} = \frac{\upsilon \sigma_t}{\sum_i \frac{\frac{1}{u_\varepsilon^i}\mu_i}{\sum_j \frac{1}{u_\varepsilon^j}\mu_j} u_{ct}^i}. \tag{22}$$

Each household receives a rebate of their tax bill so that individual transfers equal $T_t^i = \tau_t^{\mathbf{N}} c_{dt}^i$ for every period t.

Relative to a utilitarian planner, the Negishi planner has less concern for the plight of poor households. Consequently, the Negishi planner has a lower social value of consumption and taxes carbon emissions more aggressively.

4.2 The Constrained-Efficient Carbon Tax

Next, we shift our attention to a utilitarian planner that weights households equally according to their population measures. This planner would like to redistribute income so we restrict it to choosing allocations that imply no resource transfers across households, thereby forcing it to preserve initial inequality. We will show that, when marginal utility is sufficiently convex, the constrained-efficient utilitarian carbon tax differs from both the utilitarian carbon tax and the Negishi carbon tax, producing a very different distribution of welfare.

An Inequality-Neutral Climate Policy. Consider a government that rebates the proceeds from carbon taxation back to each household according to its tax bill. This transfer scheme distorts the consumption bundles of households while leaving the underlying distribution of resources unchanged.

Specifically, transfers are equal to

$$T_t^i = \tau_t^i c_{dt}^i \tag{23}$$

for all i and t. Plugging the transfer scheme (23) into the household budget constraint, the planner is now constrained to consider only allocations that satisfy the following implementability condition:

$$c_{ct}^i + c_{dt}^i \leq \varepsilon^i \tag{24}$$

for all i and t.

Condition (24) is certainly more restrictive than the feasibility condition (10) and prevents the utilitarian planner from pursuing any direct redistribution. The constrained-efficient carbon tax and transfer scheme arise from implementing the constrained-efficient allocation as a competitive equilibrium.

Definition 3 (Constrained-Efficient Allocation) *The constrained-efficient allocation is the sequence $\{c_{dt}^i, c_{ct}^i, S_t\}_{t=0}^{\infty}$ that solves the constrained-efficient utilitarian social planner's problem, which is to maximize social welfare (9), with $\alpha_i = \mu_i$ for all i, subject to the carbon cycle (1) and the implementability condition* (24).

The first-order conditions for this problem are:

$$(c_{dt}^i) : u_{dt}^i - \upsilon\sigma_t - \lambda_t^i = 0 \tag{25}$$

$$(c_{ct}^i) : u_{ct}^i - \lambda_t^i = 0 \tag{26}$$

$$(S_{t+1}) : -x'(S_{t+1}) + \sigma_t - \beta\sigma_{t+1}(1-\delta) = 0 \tag{27}$$

where $\beta^t \mu_i \lambda_t^i$ is the Lagrange multiplier on the implementability condition (24).

Combine equations (25) and (26) to obtain:

$$\frac{u_{dt}^i}{u_{ct}^i} = 1 + \frac{\upsilon\sigma_t}{u_{ct}^i} \tag{28}$$

In contrast to the unconstrained optimal allocation, the marginal utilities in (25) and (26) and the marginal rate of substitution between consumption of clean and dirty goods in (28) are not equal across all households. It follows that the constrained-efficient carbon tax is no longer uniform. The following proposition characterizes the constrained-efficient carbon tax. The proof is in Appendix C.

Proposition 1 (Constrained-Efficient Carbon Tax) *Suppose that the constrained-efficient allocation is $\{c_{dt}^i, c_{ct}^i, S_t\}_{t=0}^{\infty}$ for all i. Then, there exists a sequence of prices $\{w_t, p_t\}_{t=0}^{\infty}$ such that the allocation is a competitive equilibrium with taxes given by*

$$\tau_t^i = \frac{\upsilon\sigma_t}{u_{ct}^i} \quad \forall i. \tag{29}$$

The tax revenue is rebated back to each household with transfers equal to $T_t^i = \tau_t^i c_{dt}^i$ for every period t and for all i.

The constrained-efficient carbon tax in (29) is set to the social cost of carbon, valued in units of the consumption good for each household. Thus, the constrained-efficient carbon tax of a heterogeneous economy is itself heterogeneous. For each household, the social cost of carbon is valued at its own marginal utility. That is, even though all households share the same climate damages in absolute terms, they differ in how much they value the climate damages relative to their own consumption. Because households with lower income have a higher marginal utility, it follows that

$$\tau_t^j < \tau_t^k$$

for all j and k with $\varepsilon^j < \varepsilon^k$.

Therefore, the constrained-efficient carbon tax calls for a higher rate for households with higher incomes. Notice that, absent any transfer of resources across households, some redistribution of welfare still occurs indirectly through the differential tax rates. In this way, the carbon tax serves a dual role of achieving efficiency by correcting the climate externality while respecting equity considerations.

4.3 Constrained-Efficient Uniform Carbon Tax

A uniform carbon tax is the rule considered in most policy proposals. However, it is not the optimal tax rate in a heterogeneous economy when transfers across households are ruled out. To recover a uniform carbon tax, uniformity of the tax rate must be imposed as an additional constraint in the planning problem. From (8), this constraint expressed in terms of the allocation is given by:

$$\frac{u_{dt}^i}{u_{ct}^i} = \frac{u_{dt}^j}{u_{ct}^j} \tag{30}$$

for all i, j. Furthermore, for preferences of the form

$$u(c_{ct}, c_{dt}) = \frac{\left((c_{ct} + \bar{c})^{\gamma} \ c_{dt}^{1-\gamma}\right)^{1-\kappa}}{1-\kappa} \tag{31}$$

the constraint can be written as

$$\left(c_{ct}^i + \bar{c}\right) c_{dt}^j = \left(c_{ct}^j + \bar{c}\right) c_{dt}^i \tag{32}$$

for all i, j. The parameter γ represents preference over clean consumption and $\bar{c} > 0$ is the non-homotheticity parameter, which allows the model to match the differences in embodied emission intensities across households documented in Section 2. Additionally, we assume that $\kappa > 1$.

The following proposition characterizes the constrained-efficient climate policy in an economy where the planner is fully constrained from using climate policy to redistribute resources across households. In the model, this restriction implies no direct transfer of resources across individuals and uniform carbon taxes. The proof can be found in Appendix C.

Proposition 2 (Constrained-Efficient Uniform Carbon Tax) *Suppose that the allocation* $\{c_{dt}^i, c_{ct}^i, S_t\}_{t=0,\forall i}^{\infty}$ *solves the constrained-efficient planner's problem with the additional constraint* (32). *Then, there exists a sequence of prices* $\{w_t, p_t\}_{t=0}^{\infty}$ *such that the allocation is a competitive equilibrium with taxes given by*

$$\tau_t = \frac{\upsilon \sigma_t}{\sum_i \frac{\mu_i c_t^i}{\sum_j \mu_j c_t^j} u_{ct}^i} \tag{33}$$

with $c_t^i \equiv c_{ct}^i + \bar{c} + c_{dt}^i$*. The revenue is rebated back with transfers equal to* $T_t^i = \tau_t c_{dt}^i$ *for every period* t *and for all* i*.*

The constrained-efficient uniform carbon tax follows the Pigouvian rule, by using a weighted average of marginal utilities to price the climate externality. When the marginal utility function is sufficiently convex, climate mitigation under a uniform constrained planner is moderated relative to the utilitarian optimal benchmark. The intuition for this result is simple. Because the weighted average of marginal utilities is greater than the marginal utility of average consumption, the social value of consumption is higher for the constrained planner. For a given marginal social damage, this will result in a lower carbon tax relative to both the utilitarian optimal tax and the Negishi optimal tax, provided that the risk aversion κ is greater than one. In fact, when $\kappa = 1$, the formulas for the utilitarian optimal carbon tax, Negishi optimal carbon tax, and uniform constrained optimal tax are identical, and in particular, the Negishi optimal and uniform constrained-efficient allocation and tax rates are identical.

Two central takeaways arise from the analysis. First, uniform carbon taxes, considered in most policy proposals, are not optimal in an unequal world. The constrained optimal planner understands that high-income individuals place more value on the climate relative to consumption (because the marginal utility of consumption is low for these individuals) and assigns a higher tax relative to low-income households, which place a higher value on consumption.

The second takeaway from our analysis highlights the implications of explicitly imposing uniform taxation as a policy restriction. In an economy with heterogeneity, the constrained-efficient carbon tax formula differs from the unconstrained optimal carbon tax and takes into account the distribution of consumption. In this case, the social cost of carbon is priced at a weighted average of households' marginal utilities, which is higher than the marginal utility of the representative agent when marginal utilities are sufficiently convex, leading to a lower carbon tax.

Alternative Decentralization. The implementation of the uniform-constrained carbon tax with individual lump-sum rebates can be challenging when the planner lacks enough information. In the next result, we characterize an alternative all-uniform climate policy that consists of a uniform carbon tax, a uniform clean subsidy, and a uniform transfer. In particular, consider an alternative market economy where households face a carbon tax on dirty consumption, τ_{dt}, a clean subsidy, τ_{ct}, and lump-sum transfers, T_t. The household's problem is to maximize (2) subject to the following set of budget constraints,

$$p_t(1+\tau_{dt})c^i_{dt} + (1-\tau_{ct})c^i_{ct} \leq w_t\varepsilon^i + T_t \tag{34}$$

for every period t. The first-order conditions for this problem lead to the following optimality condition:

$$\frac{u^i_{dt}}{u^i_{ct}} = \frac{1+\tau_{dt}}{1-\tau_{ct}} \tag{35}$$

where the marginal rate of substitution between clean and dirty consumption equals the relative after-tax price of the goods. As before, optimality on the firm's side implies that $p_t = w_t$.

Corollary 1 (Uniform Carbon Tax, Clean Subsidy, and Transfer) *Suppose that the allocation* $\{c^i_{dt}, c^i_{ct}, S_t\}^{\infty}_{t=0,\forall i}$ *solves the constrained-efficient planner's problem with the additional constraint* (32)*. Then, the constrained-efficient allocation* $\{c^i_{dt}, c^i_{ct}, S_t\}^{\infty}_{t=0,\forall i}$ *is also implementable as a competitive equilibrium with an all-uniform climate policy* $\{\tau_{dt}, \tau_{ct}, T_t\}$ *given by:*

$$\tau_{dt} = \gamma\tau_t \quad ; \quad \tau_{ct} = (1-\gamma)\frac{\tau_t}{1+\tau_t} \quad ; \quad T_t = \tau_{ct}\bar{c} \tag{36}$$

where τ_t *is given by* (33) *from Proposition 2.*

The proof is in Appendix C. This alternative policy can implement the constrained-efficient allocation, providing an arguably more viable alternative to the uniform-constrained carbon tax with individual rebates.

4.4 A Numerical Illustration

In this section, we numerically illustrate the welfare and climate consequences of the various optimal tax rules characterized in Sections 4.1–4.3. We assume that the labor endowments, ε^i, are distributed according to a log normal distribution, parameterized to match the consumption inequality of the quantitative model introduced in the next section. The remaining parameters are chosen to align with observed moments related to emissions embedded in household expenditures, total global emissions, and estimates of climate damages.

Table 3 reports the welfare and climate consequences of implementing the heterogeneous constrained-efficient tax from Proposition 1, the uniform constrained-efficient carbon tax from Proposition 2, and the Negishi optimal tax from equation (22). These three cases represent an apples-to-apples comparison, as none of them involve net transfers across households. The numerical exercise has two main takeaways. First, the heterogeneous constrained-efficient carbon tax delivers the highest average welfare gain and largest long-run temperature reduction, though households in the top decile of the income distribution suffer a welfare loss, due to the very distortionary carbon taxes levied on them.

Table 3: Welfare and Climate

Policy	Initial carbon tax ($/ton)	Long-run temp. reduction (degrees)	Average welfare gain (percent)	Support (percent)
Constrained-efficient tax	111.7*	1.9	0.068	90.0
Uniform constrained-efficient tax	54.3	1.2	0.063	100.0
Negishi optimal tax	89.9	1.7	0.060	76.0

Note: The model is parameterized as follows: $\log \varepsilon \sim \mathrm{N}(0, 0.35)$, $x(S) = \Psi/2X^2$, $\gamma = 0.98$, $\bar{c} = 0.14$, $\Psi = 0.03$, $\beta = 0.97$, $\kappa = 2$, $\delta = 1/300$, and $\upsilon = 326.4$. * denotes the average carbon tax rate, weighted by dirty consumption expenditures.

Second, if we compare the two uniform carbon taxes, one in which the utilitarian planner is constrained from transferring resources across households (uniform constrained-efficient), and the other in which the planner weights households in such a way that there is no incentive to transfer resources across households (Negishi optimal), the uniform constrained-efficient tax delivers not only higher welfare gains on average, but is also the only policy that is Pareto improving. By putting higher weight on high-income households, which value consumption less than low-income households, the Negishi planner sets a carbon tax that is higher than what most low-income households would prefer, resulting in a welfare loss for the bottom quartile of the income distribution. For this and other technical reasons, we henceforth focus on the utilitarian constrained optimal taxes.[5]

In the next section, we conduct a quantitative exploration of the utilitarian constrained-efficient tax rates in an extended quantitative model.

[5] In the next section, household permanent labor endowments are replaced by sequences of uninsurable, idiosyncratic labor productivity shocks. Properly formulating the Negishi optimal tax in that environment would require the Negishi weights to be time-varying.

5 Quantitative Analysis

5.1 Quantitative Model

For the quantitative analysis, we extend the baseline model to incorporate endogenous labor and savings decisions, borrowing constraints, and a more comprehensive fiscal policy. This extension aims to create an empirically grounded economy that reflects existing inequality and policy instruments. The quantitative model allows us to quantify both the effectiveness of carbon taxes in addressing climate externalities and their distributional impacts.

In this version of the economy, households face idiosyncratic labor productivity risk, which is uninsurable. We assume that labor productivity ε_t^i follows a Markov process with transition matrix $\pi\left(\varepsilon_t^i, \varepsilon_{t+1}^i\right)$. Households supply $n_t^i \varepsilon_t^i$ efficiency units of labor, where n_t^i denotes hours worked.

There is no aggregate uncertainty. Households can save in the form of real capital, k_{t+1}^i, which depreciates at a constant rate, δ_k. We assume households face the borrowing constraint

$$k_{t+1}^i \geq \underline{a}, \tag{37}$$

for every period t.

Households face capital income taxes, τ_{kt}, and a nonlinear labor income tax, $T_t^n(w_t \varepsilon_t^i n_t^i)$. In addition, they pay a carbon tax, τ_t^i, and receive lump-sum transfers, T_t^i. Thus, the household's budget constraint for every period t is

$$p_t(1+\tau_t^i)c_{dt}^i + c_{ct}^i + k_{t+1}^i \leq w_t \varepsilon_t^i n_t^i - T_t^n(w_t \varepsilon_t^i n_t^i) + (1-\tau_{kt})r_t k_t^i + (1-\delta_k)k_t^i + T_t^i \tag{38}$$

where w_t is the wage per efficiency unit of labor. The price of the clean good is normalized to one.

The household's problem is to choose consumption, labor, and savings to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t [u(c_{ct}^i, c_{dt}^i) - v(n_t^i) - x(S_{t+1})] \tag{39}$$

subject to (37) and (38).

The production of the clean and the dirty consumption good uses labor and capital as inputs according to a constant return to scale technology. Aggregate production of the clean and the dirty good is given by $Y_{ct} = F(K_{ct}, N_{ct})$ and $Y_{dt} = F(K_{dt}, N_{dt})$, respectively. The problem of the producer is to choose $\{\{N_{jt}, K_{jt}\}_{j=c,d}\}_{t=0}^{\infty}$ to maximize profits.

The government collects taxes and uses the proceeds to finance government spending and provide transfers, so that

$$\sum_i \mu_i \left(\tau_t^i p_t c_{dt}^i + T_t^n(w_t \varepsilon_t^i n_t^i) + \tau_{kt} r_t k_{t+1}^i\right) = G_t + \sum_i \mu_i T_t^i. \tag{40}$$

Finally, market clearing for each period t requires that

$$N_{ct} + N_{dt} = \sum_i \mu_i \varepsilon_t^i n_t^i \tag{41}$$

$$K_{ct} + K_{dt} = \sum_i \mu_i k_t^i \tag{42}$$

$$\sum_i \mu_i \left(c_{ct}^i + k_{t+1}^i - (1-\delta_k)k_t^i\right) + G_t = F(K_{ct}, N_{ct}) \tag{43}$$

$$\sum_i \mu_i c_{dt}^i = F(K_{dt}, N_{dt}) \tag{44}$$

Definition 4 *A competitive equilibrium with taxes* $\{\tau_t^i, \tau_{kt}, T_t^n, T_t^i\}_{t=0}^\infty$ *is a sequence of prices* $\{p_t, w_t, r_t\}_{t=0}^\infty$ *and allocations* $\{c_{ct}^i, c_{dt}^i, n_t^i, k_t^i, \{N_{jt}, K_{jt}\}_{j=c,d}\}_{t=0}^\infty$ *such that (i) given prices, households choose* $\{c_{ct}^i, c_{dt}^i, n_t^i, k_t^i\}_{t=0}^\infty$ *to maximize (39) subject to (38) and (37) for all i; (ii) profit maximizing prices are* $w_t = F_{Njt}$, $r_t = F_{Kjt}$ *for* $j = c, d$ *and* $p_t = 1$*; (iii) the stock of atmospheric carbon evolves according to* (1)*; (iv) the government budget constraint* (40) *is satisfied; and (v) markets clear,* (41)–(44).

We use the model to study the implications of implementing a carbon tax in an empirically motivated economy designed to reflect existing inequality and taxes. The model economy targets moments of the US economy and its tax system and scales up emissions to account for the global carbon stock. We describe the calibration strategy below, summarized in Table 4.

5.2 Calibration

Economic Parameters. Because high-income, high-wealth households account for the bulk of consumption and, therefore, also of emissions, it is important that the model generates a distribution that is skewed in both of these dimensions. To achieve this, we employ a common strategy from the literature and include a superstar state in the Markov chain for the productivity process (Castaneda et al., 2003). To calibrate this Markov chain, we first approximate an AR(1) process (in logs) using the Rouwenhorst method (Kopecky and

Table 4: Calibration

Parameters	Values	Targets / Source
Preferences		
Discount factor, β	0.97	capital-to-output: 4.8
Risk aversion, κ	2	standard value
Disutility from labor, ϕ	30	average hours: 30 percent
Frisch elasticity, $1/\nu$	0.50	standard value
Climate		
Carbon absorption, δ	1/300	average life of carbon: 300 years
Carbon intensity, υ	326	1.4 degree increase by 2100 under BAU
Utility loss, ψ	0.04	welfare loss from 2.5 degree increase $\approx$ 1.74 percent output reduction
Clean share, γ	0.97	\$50/ton carbon tax leads to 0.8 degree reduction from BAU
Nonhomotheticity, $\bar{c}$	0.16	emissions intensity 31 percent higher for low-income than high-income households
Taxes		
Average, τ_n	0.23	average net tax rate: 13 percent
Progressivity, ν_y	0.17	37.9 percent marginal tax rate on top 1 percent earner
Capital, τ_k	0.27	Carey and Rabesona (2002)
Technology and shocks		
Capital weight, α	0.36	capital income share: 36 percent
Capital depreciation rate, δ_k	0.05	standard value
Persistence of wage process, ρ	0.94	author estimates
Standard deviation, σ_ε	0.24	Gini coefficient of earnings: 0.47
Superstar productivity, ε_{sup}	162.6	wealth share of top 1%: 34%
Persistence of superstar state, $\pi_{10,10}$	0.94	Gini coefficient of wealth: 0.83
Superstar probability, $\pi_{1:9,10}$	6e-5	fraction of superstars: 0.1%

Suen, 2010) with nine normal (i.e., non-superstar) states. The persistence of the process for these states is set to 0.94 as measured in the PSID. Next, we jointly calibrate the standard deviation of the normal process, the value of superstar productivity, and the persistence of the superstar state to target three additional moments from the data: a Gini coefficient of earnings of 0.47, a top 1 percent wealth share of 0.34, and a Gini coefficient of wealth of 0.83. The probability of becoming a superstar from any normal state is set so that superstars account for 0.1 percent of the population. When a household exits the superstar state, its new productivity level is drawn from the ergodic distribution over the normal states as in Boar and Midrigan (2022). We set the borrowing limit to $\underline{a} = 0$ so that the model generates a share of households with non-positive wealth that is similar to that in the data.

The utility function is as specified in (31), and its preference parameters are calibrated to reflect the greater carbon intensity in the consumption baskets of low-income households as documented in Section 2. Accordingly, the nonhomotheticity parameter, $\bar{c}$, is set so that emissions intensity is 31 percent higher for households in the bottom 10 percent of income compared to those in the top 10 percent. We assume the disutility of labor takes the form

$$v(n) = \phi \frac{n^{1+\nu}}{1+\nu}, \tag{45}$$

where ϕ and ν govern the disutility of labor and the Frisch labor elasticity, respectively.

Climate Parameters. We follow Golosov et al. (2014) in assuming that the stock of atmospheric carbon affects temperature changes according to:

$$T_t = \frac{\lambda}{\log(2)} \log\left(\frac{S_t}{\overline{S}}\right), \tag{46}$$

where $\bar{S} = 581$ represents the pre-industrialization carbon stock (in gigatons) and $\lambda = 3$, which implies that for each doubling of the carbon stock the temperature increases by 3 degrees (Celsius). We set $S_{2023} = 785$ to match the temperature rise of 1.3 degrees from the pre-industrial mean.

The carbon disutility cost takes the form

$$x(S) = \frac{\Psi}{2} S^2, \tag{47}$$

which is equivalent to the formulation in Barrage (2020) when risk aversion κ is equal to 2.[6] We calibrate Ψ so that the welfare loss associated with a 2.5-degree temperature increase is

[6] In Appendix D, we explore an extension of the model to heterogeneous climate damages and discipline the sensitivity with respect to income using survey data. We find that the main results are robust to this alternative specification.

equivalent to that from a 1.74 percent decline in output, which combines the production and utility damages used in Barrage (2020).

We calibrate consumption carbon content, υ, so that under a business-as-usual scenario, there is an additional 1.4 degree increase in temperature from 2023 to 2100 (for a total of a 2.7 degree increase from pre-industrial levels).[7] We set the rate of natural reabsorption, δ, to 1/300 so that the average life cycle of carbon is 300 years (Archer 2005). Finally, we set the dirty consumption share in the utility function, $1-\gamma$, so that a \$50/ton carbon tax leads to 0.8 degree reduction in global temperature from business-as-usual, consistent with Krusell and Smith Jr. (2022).

Taxes. We take the US tax structure as given, including its progressive earnings tax and transfer system and capital income tax. Thus, the BAU scenario corresponds to the US economy with its current tax regime and no carbon taxation. Following Heathcote et al. (2017) and Holter et al. (2019), the earnings tax bill for a household with pre-tax earnings $y = w_t n_t^i \varepsilon_t^i$ takes the form

$$T_t^n(y) = y - \tilde{y}^{\nu_y} \frac{1-\tau_n}{1-\nu_y} y^{1-\nu_y} \tag{48}$$

where $\tilde{y}$ denotes average earnings in the economy. The parameter τ_n shifts the average tax rate, while ν_y controls the progressivity of the tax schedule. A flat labor income tax, τ_n, corresponds to $\nu_y = 0$. As ν_y increases, the tax function becomes more progressive. The capital income tax, τ_k, is set to the average rate for the US (Carey and Rabesona, 2002).

5.3 Quantitative Results

We use the calibrated model to quantify the aggregate and distributional effects of different carbon tax policies on both the economy and the climate. The carbon tax formulas we derived in Section 4 are the theoretical foundation for our quantitative analysis. Specifically, we evaluate the formulas for the heterogeneous tax rates (29) and the uniform tax rate (33) at the competitive equilibrium allocation to determine the path of the carbon taxes. This requires searching for a fixed point in the space of carbon tax sequences through a simple iterative strategy. Starting from a sequence with zero carbon taxation (i.e., business-as-usual), we compute the equilibrium paths for the carbon stock, the wealth distribution, and household consumption decisions. Next, we plug this allocation into the carbon tax formula to generate a new sequence of carbon taxes. Then we solve for the transition corresponding to this updated sequence and evaluate the tax formula again. We continue to iterate on this

[7] See `https://climateactiontracker.org/global/cat-thermometer`.

procedure, repeatedly updating the path of carbon taxes until it converges.[8] Unless otherwise noted, the carbon tax policy rebates each household's carbon tax bill back to it.

We begin by contrasting the outcomes of two carbon tax policies: one in which the government levies the same flat rate, τ_t, on all households according to the uniform constrained-efficient carbon tax formula in (33), and a second one, in which the carbon tax schedule places higher tax rates on more productive households as in (29).[9] Figure 2 plots the time path of the uniform and heterogeneous optimal carbon tax schedules. In both cases, the tax rates rise over time, reflecting that the greatest climate damages appear in the future and are thus heavily discounted in the initial periods. As time passes, however, and atmospheric carbon levels rise, the benefits of discouraging additional carbon emissions become more pressing.

Along the uniform tax path (shown in panel (a)), the carbon tax rate starts at \$41/ton and gradually increases to a long-run value of \$78/ton. When the carbon tax schedule differentiates households by labor productivity (effectively a household's hourly wage), the tax rates display a significant degree of variation. The tax rate on the lowest productivity households rises from \$10/ton in the initial period to \$23/ton in the long run. In contrast, the analogous rates for households with the highest non-superstar productivity are \$190/ton and \$445/ton. The wide difference in tax rates reflects the sizeable variation in the marginal utility of consumption across households with different labor productivity. It is not surprising then that the carbon tax rate for superstar households is extraordinarily high, starting at \$8,400/ton and rising to nearly \$20,000/ton.

Because we have shut off the wealth effect by rebating back each household's carbon tax payment, the aggregate levels of labor, capital, consumption, and output remain very close to their initial values under either policy. The composition of these aggregates between dirty and clean goods, however, does change since the tax distorts each household's optimal consumption bundle toward a higher share of clean consumption.

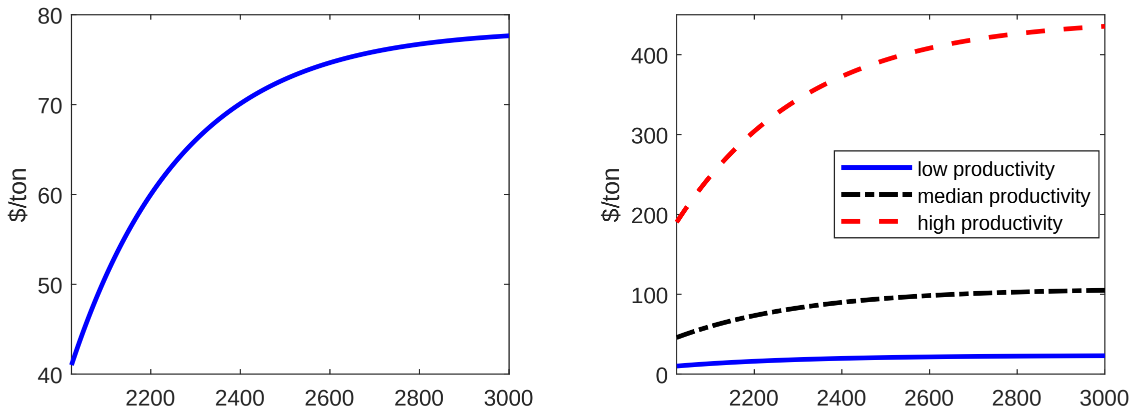
Turning to the climate, global temperature still rises under either carbon tax policy. Nevertheless, relative to the BAU scenario, both fiscal interventions have a substantial effect on the evolution of the carbon stock and global temperature over time (Figure 3). The greatest temperature moderation is produced by the productivity-indexed carbon tax. It

[8] In Appendix B, we formally show that these carbon taxes are part of the constrained-efficient fiscal policy of the quantitative economy, which also includes income taxes due to the pecuniary externality that arises from the presence of uninsurable idiosyncratic productivity risk and exogenous borrowing limits. In this paper, we focus on addressing the climate externality and thus leave income taxes unchanged.

[9] We index the heterogeneous tax rates by productivity. While taxes could in principle depend on the entire history of a household's productivity shocks, this is computationally infeasible. Alternatively, indexing by wealth would introduce a distortion to the savings decision.

Figure 2: Constrained-efficient carbon tax

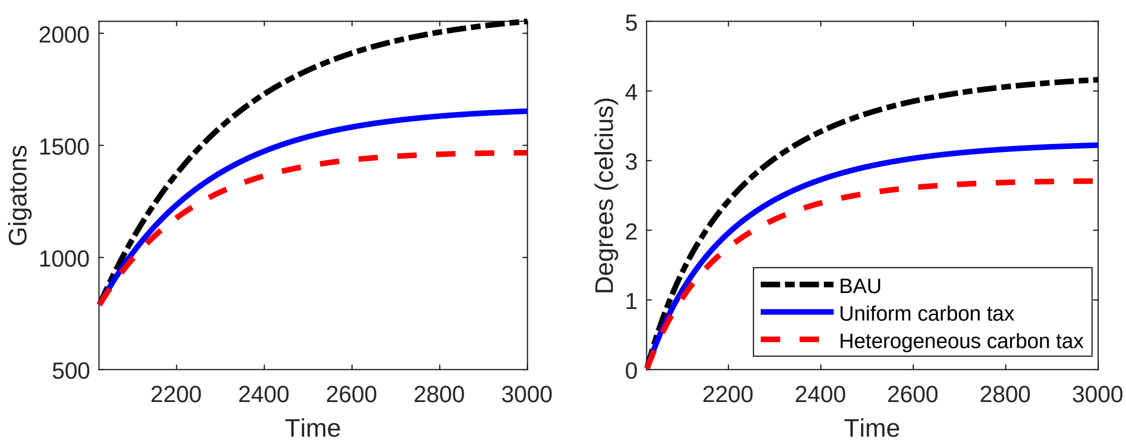
(a) Uniform (b) Heterogeneous



reduces global temperatures by 0.5°C relative to the BAU path over 100 years and by 1.2°C over 500 years.

Figure 3: Carbon and temperature

(a) Carbon stock (b) Temperature increase



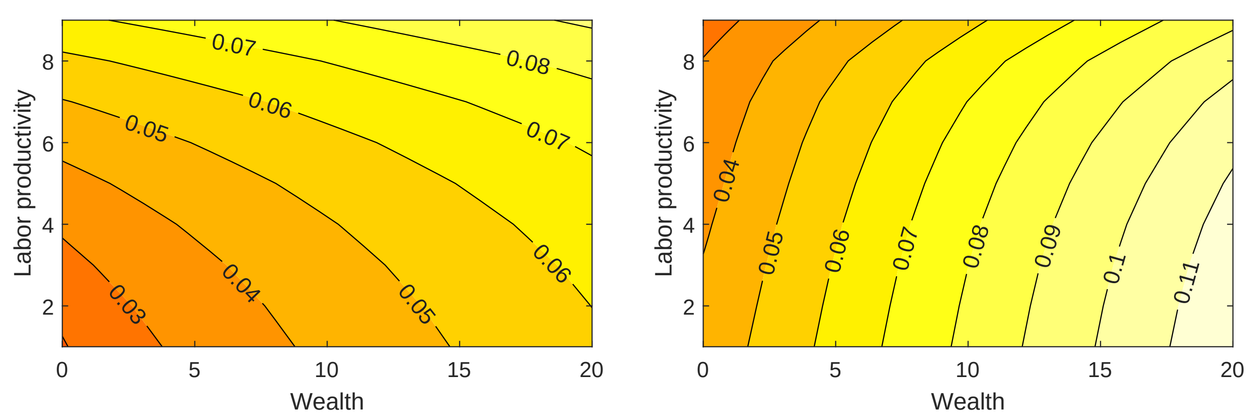
Next, we compute the change in welfare from undergoing the policy-induced transitions relative to the BAU baseline and examine the differential effects of the carbon tax across the wealth and income distribution. The welfare change is measured from the perspective of agents at $t = 1$ and includes the entire discounted sequence of utility differences.

Figure 4 displays the change in welfare for households across the wealth and productivity distribution under carbon taxation. The wealth levels shown cover 98 percent of households. In panel (a), where carbon taxes are uniform, all households experience welfare gains, with the largest gains accruing to the most productive households with high wealth. While all households benefit from reduced emissions, the costs, particularly from distortions in consumption patterns, disproportionately affect poorer households. This is further illustrated in the heterogeneous tax case (panel b), where some of these distortions are shifted from low- to high-productivity households. In this case, welfare gains are decreasing in productivity, for a fixed level of wealth. In fact, a small fraction of the highest-productivity households initially suffer a small welfare loss due to the steep taxes imposed on their consumption of carbon-intensive goods.

Figure 4: Welfare (consumption equivalents, percent)

(a) Uniform

(b) Heterogeneous



There is a timing mismatch between the costs and benefits of taxing carbon emissions. The consequences of unmitigated carbon emissions build up over time, with the most severe effects from the business-as-usual scenario materializing in the distant future. Meanwhile, any carbon policy stringent enough to have a meaningful impact on the path of the carbon stock must impose immediate costs on households. The balance between these costs and benefits shifts over time. To illustrate, we compute average welfare over time by integrating welfare changes from time t forward against the income and wealth distribution. For each period t_n, $n = 1, ..., T$, this measures the permanent increase in consumption that households require to be indifferent between being dropped into period t_n of the BAU transition path or being dropped into t_n on the climate policy transition path (i.e., where a carbon tax was enacted starting in $t = 1$). Figure 5 plots the evolution of average welfare, measured as consumption equivalents under the veil of ignorance, and decomposes welfare changes into economic and climate factors. Although we do not explicitly model overlapping generations, one could also interpret this as a proxy measure for the average welfare of future generations. Over time, the benefits from a better climate grow, while the costs associated with consumption distortions remain relatively stable.

Figure 5: Average welfare over time

(a) Uniform (b) Heterogeneous

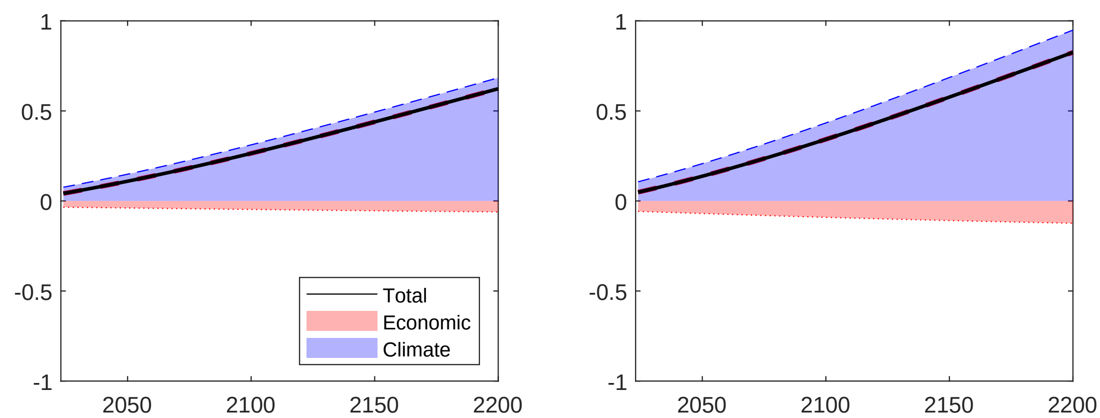


Table 5 is the quantitative model analogue of Table 3 from the simple model, and it reports the initial carbon tax, climate consequences summarized as the long-run reduction in temperature, average initial welfare gain, and support for the policy. As in the simple model, the uniform carbon tax (lump-sum rebated back to households) delivers slightly lower average welfare gains than the heterogeneous carbon tax. Both policies have a very high amount of support. The uniform carbon tax is Pareto improving relative to BAU, while in the heterogeneous case all households benefit except some superstars. The result that the uniform carbon tax is Pareto improving is robust to alternative parameter values and model assumptions. Details are in Appendix D.

Table 5: Welfare and Climate

Policy	Initial carbon tax ($/ton)	Long-run temp. reduction (degrees)	Average welfare gain (percent)	Support (percent)
Heterogeneous tax with rebate*	168.2	1.6	0.052	99.9
Uniform tax with rebate	41.0	1.0	0.045	100.0
Uniform tax & subsidy/transfer	41.0	1.0	0.047	100.0
Constant tax with rebate	100.0	1.2	0.008	49.7

Note: * denotes the average carbon tax rate, weighted by dirty consumption expenditures.

We also report the outcome of two other policies. The first imposes a uniform carbon tax and then divides the revenue between financing a subsidy on clean goods and a uniform lump-sum transfer as described in Corollary 1 (shown in row 3). The welfare and climate consequences under this policy are nearly identical to those from the uniform carbon tax with individual rebates (row 2). While Corollary 1 established an equivalence in the simple model, this result was not guaranteed to hold in the more general quantitative model studied in this section. This is an important result, since the all-uniform tax/subsidy/transfer policy does not require the policymaker to have knowledge of each household's income or wealth, making it easier to implement.

For the final case in Table 5, we impose a simple carbon tax of $100/ton that is uniform over households and constant over time. This leads to much lower welfare gains and support (row 4) compared to either time-varying uniform carbon tax policy (rows 2 and 3), highlighting the importance of our theory to informing carbon tax design.

6 Concluding Remarks

In this paper, we study the design of optimal climate policy in the presence of economic inequality. Our analysis yields several key theoretical insights into the implications of incorporating inequality into climate policy design: (*i*) a heterogeneous carbon tax emerges as a progressive natural solution, where higher-income households face higher tax rates; (*ii*) if uniformity of the carbon tax is desired, the carbon tax is lower than that prescribed under a representative agent framework; and (*iii*) uniform taxation can be achieved using a combination of three instruments: a uniform carbon tax, a uniform clean energy subsidy, and a uniform lump-sum transfer. Our quantitative analysis suggests that the uniform climate policy is welfare-improving for every household.

Our work suggests several promising directions for future research. First, while this paper focuses on consumption inequality, income inequality may also arise from jobs that are more vulnerable to climate shocks, such as agriculture, or from sectors more heavily affected by climate regulation, such as energy. As a result, climate inequality may be linked to employment in sectors that are more exposed to climate change. Endogenizing the unequal effects of climate change by enriching the modeling of the economy's productive sectors presents a significant avenue for further research. Second, households' climate inequality stems also from heterogeneous climate impacts, with the concern that low-income households might be more vulnerable to climate shocks. While we show that our results remain robust when utility costs are heterogeneous by income, further research quantifying this aspect of climate vulnerability would be beneficial. Finally, while this paper examines within-country income and wealth inequality, multiple dimensions of inequality are crucial to the climate problem, including cross-country inequality and intergenerational inequality. Exploring the policy implications of these alternative dimensions would provide valuable insights for climate policy design.

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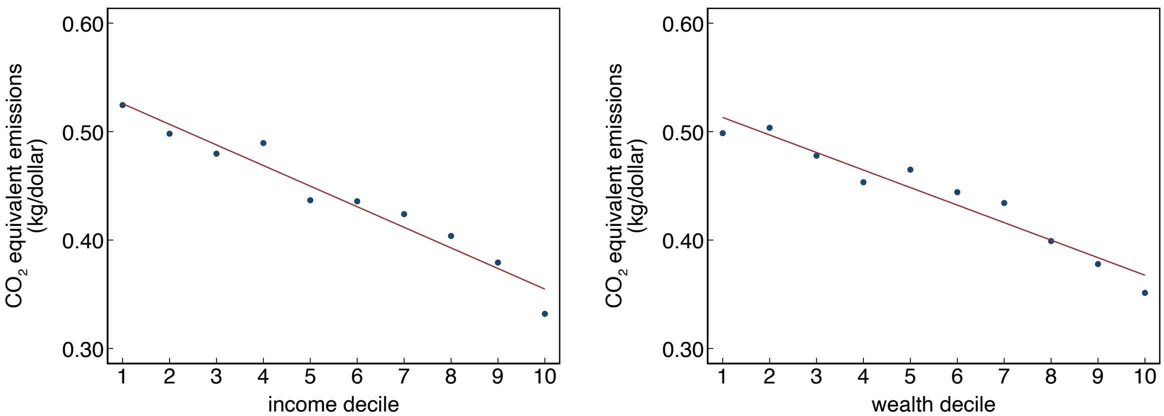
A Data Appendix

We redo the empirical analysis using household data from the Panel Study of Income Dynamics (PSID). Compared to the CEX, the PSID has the advantage that it contains a more complete representation of household wealth, including financial and nonfinancial assets and debt. On the other hand, the PSID expenditure data are more aggregated, compared with the CEX. Thus, the PSID analysis presented here complements our CEX analysis and provides a useful robustness exercise.

We combine the PSID expenditure data with the EPA emissions data, similar to the way described in Section 2. As shown in Figure 6, the embodied emissions intensity is decreasing in both income and in wealth.

Figure 6: Embodied emissions

(a) Income (b) Wealth



B Constrained Efficiency in the Quantitative Model

This section shows that the carbon tax formulas we derive in the simple economy go through to the richer quantitative economy. As demonstrated in Dávila et al. (2012), the economy with incomplete markets is typically not constrained-efficient and can be improved by a utilitarian planner. This is because market incompleteness causes household labor and savings decisions to affect the equilibrium prices of labor and capital—an equilibrium effect that individuals do not internalize. Here, this so-called pecuniary externality adds to the climate externality in our economy.

Consider the model environment laid out in Section 5. The *constrained-efficient allocation* is the sequence $\{\{c^i_{ct}, c^i_{dt}, n^i_t, k^i_{t+1}\}_i, \{K_{jt}, N_{jt}\}_{j=c,d}, S_t\}_{t=0}^{\infty}$ that solves the social planner's problem, which is to maximize

$$\mathbf{E}_0 \sum_i \mu_i \left[\sum_{t=0}^{\infty} \beta^t \left(u(c^i_{ct}, c^i_{dt}) - v(n^i_t) - x(S_{t+1}) \right) \right], \tag{49}$$

subject to the carbon cycle (1), household borrowing constraints (37), and implementability conditions

$$c^i_{dt} + c^i_{ct} + k^i_{t+1} \leq F_{Nt} \varepsilon^i_t n^i_t + F_{Kt} k^i_t + (1 - \delta_k) k^i_t. \tag{50}$$

The implementability conditions can be derived by substituting equilibrium prices $w_t = F_{Nt}$, $r_t = F_{Kt}$, and $p_t = 1$, and the constraint that individual taxes must be rebated lump-sum into each household's budget constraint (38), with $G_t = 0$. The constrained-efficient allocation takes the market structure and individual constraints as given, and excludes net transfers across households.

The first-order conditions for the constrained-efficient social planner's problem are:

$$(c_{dt}^i) : \mu_i u_{dt}^i - \upsilon \mu_i \sigma_t - \mu_i \lambda_t^i = 0 \tag{51}$$

$$(c_{ct}^i) : \mu_i u_{ct}^i - \mu_i \lambda_t^i = 0 \tag{52}$$

$$(n_t^i) : -\mu_i v_{n_t}^i + \mu_i \lambda_t^i F_{Nt} \varepsilon_t^i + \sum_j \mu_j \lambda_t^j (F_{NNt} \mu_i \varepsilon_t^i \varepsilon_t^j n_t^j + F_{KNt} \mu_i \varepsilon_t^i k_t^j) = 0 \tag{53}$$

$$(k_{t+1}^i) : -\mu_i \lambda_t^i + \mu_i \phi_t^i + \beta \mu_i \mathbf{E}_t \left[\lambda_{t+1}^i (1 - \delta_k) + \lambda_{t+1}^i F_{K,t+1} \right] \tag{54}$$

$$+ \beta \mathbf{E}_t \left[\sum_j \mu_j \lambda_{t+1}^j (F_{NK,t+1} \mu_i \varepsilon_{t+1}^j n_{t+1}^j + F_{KK,t+1} \mu_i k_{t+1}^j) \right] = 0$$

$$(S_{t+1}) : \sigma_t - x'(S_{t+1}) + \beta(1 - \delta)\sigma_{t+1} = 0 \tag{55}$$

where $\beta^t \sigma_t$, $\beta^t \mu_i \lambda_t^i$, $\beta^t \mu_i \phi_t^i$ are the Lagrange multipliers on the carbon cycle, implementability conditions, and household borrowing constraints, respectively.

For ease of exposition, we define the following objects:

$$\Lambda_{nt}^i \equiv -\frac{\sum_j \mu_j u_{ct}^j (F_{NNt} \varepsilon_t^j n_t^j + F_{KNt} k_t^j)}{u_{ct}^i F_{Nt}} \tag{56}$$

$$\Lambda_{kt}^i \equiv -\frac{\sum_j \mu_j u_{ct}^j (F_{NKt} \varepsilon_t^j n_t^j + F_{KKt} k_t^j)}{u_{ct}^i (F_{Kt} + 1 - \delta_k)} \tag{57}$$

Using these definitions, we can combine the first-order conditions to get the following constrained-efficient intratemporal wedge between consumption and leisure:

$$\frac{u_{nt}^i}{u_{ct}^i} = F_{Nt} \varepsilon_t^i (1 - \Lambda_{nt}^i) \tag{58}$$

where the second term on the right-hand side captures the pecuniary externality as the social planner internalizes that household's working and savings decisions affect equilibrium factor prices, the uninsurable part of income.

Similarly, the intertemporal wedge incorporates the effect of the individual's saving decision over the equilibrium interest rate, and the Euler equation is given by:

$$u_{ct}^i \geq \beta \mathbf{E}_t \left[u_{c,t+1}^i \left(F_{Kt+1} + 1 - \delta_k \right) \left(1 - \Lambda_{k,t+1}^i \right) \right], \tag{59}$$

which holds with equality if household i's borrowing constraint is not binding.

Finally, the intratemporal wedge for clean and dirty consumption is

$$\frac{u_{dt}^i}{u_{ct}^i} = 1 + \frac{\upsilon \sigma_t}{u_{c_t}^i} \tag{60}$$

with

$$\sigma_t = \sum_{j=1}^{\infty} \left[\beta(1-\delta)\right]^{j-1} x'(S_{t+j}) \tag{61}$$

Thus, the constrained-efficient allocation equalizes the marginal rate of substitution between clean and dirty goods with the relative social price that includes the climate externality.

Proposition 3 characterizes the constrained-efficient policy that implements the optimal allocation as a market equilibrium with taxes. To derive analytical expressions for the tax rates that implement the constrained-efficient allocation, we assume that $\underline{a}$ is sufficiently low such that the borrowing constraint is not binding. Furthermore, to implement the constrained-efficient allocation, we show that the tax rates for carbon, labor income, and capital income must be individual-specific.

Proposition 3 *Let* $x \equiv \{c^i_{ct}, c^i_{dt}, n^i_t, k^i_{t+1}, \{K_{jt}, N_{jt}\}_{j=c,d}, S_t\}_{t=0}^{\infty}$ *be the constrained-efficient allocation. Then,* x *is also a competitive market equilibrium with taxes given by*

$$\tau^i_t = \frac{\upsilon \sigma_t}{u^i_{c_t}} \tag{62}$$

$$\tau^i_{nt} = \Lambda^i_{nt} \tag{63}$$

$$\tau^i_{kt} = \Lambda^i_{kt} \tag{64}$$

The tax revenue is rebated back lump-sum.

Proposition 3 shows that the carbon tax rule in the quantitative model adheres to the same formula as in Proposition 1. The actual optimal carbon tax rate, however, may differ from the rate computed in the quantitative analysis, as the calibrated economy is not constrained-optimal. Moreover, capital and labor income taxes are also part of the constrained-efficient policy. These taxes are necessary to internalize the pecuniary externalities stemming from idiosyncratic risk and market incompleteness. Notice that the constrained-efficient policy in Proposition 3 is history-dependent. This feature is also in Davila et al. (2012), making computation of the constrained-efficient taxes particularly challenging.

Uniform Constrained-Efficient Taxes. As in the simple model, uniformity of the carbon tax—if desirable—must be added as an exogenous constraint. Restricting attention to preferences of the form (31), *the uniform constrained-efficient allocation* comes from maximizing (49) subject to the carbon cycle (1), the implementability constraint (50), borrowing constraint (37), and the additional constraint (32). The first-order necessary conditions for this social planner's problem are:

$$(c^i_{dt}) : \mu_i u^i_{dt} - \upsilon\mu_i\sigma_t - \mu_i\lambda^i_t + \sum_{j\neq i}\eta^{ij}_t\left(c^j_{ct}+\bar{c}\right) - \sum_{j\neq i}\eta^{ji}_t\left(c^j_{ct}+\bar{c}\right) = 0 \tag{65}$$

$$(c^i_{ct}) : \mu_i u^i_{ct} - \mu_i\lambda^i_t - \sum_{j\neq i}\eta^{ij}_t c^j_{dt} + \sum_{j\neq i}\eta^{ji}_t c^j_{dt} = 0 \tag{66}$$

$$(n^i_t) : -\mu_i v^i_{n_t} + \mu_i\lambda^i_t F_{Nt}\varepsilon^i_t + \sum_j \mu_j\lambda^j_t(F_{NNt}\mu_i\varepsilon^i_t\varepsilon^j_t n^j_t + F_{KNt}\mu_i\varepsilon^i_t k^j_t) = 0 \tag{67}$$

$$(k^i_{t+1}) : -\mu_i\lambda^i_t + \mu_i\phi^i_t + \beta\mu_i\mathbf{E}_t\left[\lambda^i_{t+1}(1-\delta_k) + \lambda^i_{t+1}F_{Kt+1}\right] \tag{68}$$

$$+\beta\mathbf{E}_t\left[\sum_j \mu_j\lambda^j_{t+1}(F_{NKt+1}\mu_i\varepsilon^j_{t+1}n^j_{t+1} + F_{KKt+1}\mu_i k^j_{t+1})\right] = 0$$

$$(S_{t+1}) : \sigma_t - x'(S_{t+1}) + \beta(1-\delta)\sigma_{t+1} = 0 \tag{69}$$

where $\beta^t\eta^{ij}_t$ is the Lagrange multiplier on the constraint (32) and $\beta^t\sigma_t$, $\beta^t\mu_i\lambda^i_t$, $\beta^t\phi^i_t$ are the Lagrange multipliers on the carbon cycle, household implementability constraints, and borrowing constraints, respectively.

Let $x \equiv \{c^i_{ct}, c^i_{dt}, n^i_t, k^i_{t+1}, \{K_{jt}, N_{jt}\}_{j=c,d}, S_t\}^{\infty}_{t=0}$ be the solution to the uniform-constrained social planner's problem. Corollary 2 states that the constrained-efficient uniform carbon tax follows the rule in Proposition 2.

Corollary 2 *The carbon tax that implements x follows* (33)*.*

As is the case in Proposition 3, the uniform carbon tax must be coupled with individual labor and capital income taxes/subsidies to fully implement the optimal allocation.

The proofs are in Appendix C.

C Mathematical Appendix

Proof of Proposition 1. The proof consists of showing that the conditions for the constrained optimal allocation—characterized by equations (14), (24), and (28), together with feasibility constraints and the carbon cycle—satisfy the conditions of a competitive equilibrium, characterized by equations (3)–(8) with taxes τ^i_t as in (29) and transfers that satisfy (23).

First, comparing (28) and (8), simple observation shows that both coincide when τ^i_t is replaced by the optimal tax as specified in (29), $\tau^i_t = \frac{\upsilon\sigma_t}{u^i_{ct}}$ where σ_t is defined as in (14).

Second, substitute (23) into individual budget constraints in (3) with $w_t = p_t = 1$ to obtain

$$c_{ct}^i + c_{dt}^i = \varepsilon^i \tag{70}$$

Thus, (3) holds at the optimal allocation as (70) coincides with the implementability condition in (24). It is easy to check that the government's budget constraint holds, plugging (23) into (4) with $G_t = 0$ and $p_t = 1$. Finally, by (70) holding for all i and the government constraint being satisfied, market clearing conditions hold. □

Proof of Proposition 2. The proof follows from demonstrating that the equations characterizing the uniform constrained-efficient planner's problem are equivalent to the equations characterizing the competitive equilibrium, characterized by equations (3)–(8) with taxes and transfers as in (33), together with market clearing conditions.

The first-order conditions for the constrained social planner's problem are:

$$(c_{dt}^i) : \mu_i u_{dt}^i - \upsilon\mu_i\sigma_t - \lambda_t^i + \sum_{j\neq i} \eta_t^{ij}\left(c_{ct}^j + \bar{c}\right) - \sum_{j\neq i} \eta_t^{ji}\left(c_{ct}^j + \bar{c}\right) = 0 \tag{71}$$

$$(c_{ct}^i) : \mu_i u_{ct}^i - \lambda_t^i - \sum_{j\neq i} \eta_t^{ij} c_{dt}^j + \sum_{j\neq i} \eta_t^{ji} c_{dt}^j = 0 \tag{72}$$

$$(S_{t+1}) : -\beta^t x'(S_{t+1}) + \sigma_t\beta^t - \sigma_{t+1}\beta^{t+1}(1-\delta) = 0 \tag{73}$$

$$(\beta^t\lambda_t^i) : c_{ct}^i + c_{dt}^i = \varepsilon_i \tag{74}$$

$$(\beta^t\eta_t^{ij}) : (c_{ct}^i + \bar{c})c_{dt}^j = (c_{ct}^j + \bar{c})c_{dt}^i \tag{75}$$

where η_t^{ij} is the Lagrange multiplier on the constraint on allocations, (32). Iterating forward from (73), we have:

$$\sigma_t = \sum_{j=1}^{\infty} \left[\beta(1-\delta)\right]^{j-1} x'(S_{t+j}). \tag{76}$$

Combine equations (71) and (72) to obtain:

$$\frac{u_{dt}^i}{u_{ct}^i} - 1 = \frac{1}{\mu_i u_{ct}^i}\left[\upsilon\mu_i\sigma_t - \sum_{j\neq i}\eta_t^{ij}(c_{ct}^j + \bar{c} + c_{dt}^j) + \sum_{j\neq i}\eta_t^{ji}(c_{ct}^j + \bar{c} + c_{dt}^j)\right] \tag{77}$$

Multiplying both sides of equation (77) by $\mu_i u^i_{ct}(c^i_{ct}+\bar{c}+c^i_{dt})$ and sum across all i, we obtain:

$$\begin{aligned}\sum_i \left(\frac{u^i_{dt}}{u^i_{ct}}-1\right)\mu_i u^i_{ct}\left(c^i_{ct}+\bar{c}+c^i_{dt}\right) =&\upsilon\sigma_t \sum_i \mu_i\left(c^i_{ct}+\bar{c}+c^i_{dt}\right) \\ &-\sum_i\left(c^i_{ct}+\bar{c}+c^i_{dt}\right)\sum_{j\neq i}\eta^{ij}_t(c^j_{ct}+\bar{c}+c^j_{dt}) \\ &+\sum_i\left(c^i_{ct}+\bar{c}+c^i_{dt}\right)\sum_{j\neq i}\eta^{ji}_t(c^j_{ct}+\bar{c}+c^j_{dt}) \\ =&\upsilon\sigma_t\sum_i\mu_i\left(c^i_{ct}+\bar{c}+c^i_{dt}\right)\end{aligned} \tag{78}$$

where the second equality can be shown by algebra manipulation.

Equation (30) implies that the marginal rate of substitution between dirty and clean consumption must be equal for all i. Thus we can simplify equation (78) to obtain

$$\frac{u^i_{dt}}{u^i_{ct}}=1+\tau_t \tag{79}$$

where

$$\tau_t=\frac{\upsilon\sigma_t}{\sum_i\frac{\mu_i c^i_t}{\sum_j \mu_j c^j_t}u^i_{ct}} \tag{80}$$

and $c^i_t\equiv c^i_{ct}+\bar{c}+c^i_{dt}$. Comparing this equation with the market intratemporal condition (8) leads to (33).

Second, substitute $T^i_t=\tau_t c^i_{dt}$ into individual budget constraints in (3) with $w_t=p_t=1$ to obtain

$$c^i_{ct}+c^i_{dt}=\varepsilon^i, \tag{81}$$

which holds as it coincides with (24).

Finally, given that the budget constraint holds for all i and the government budget constraint holds, market clearing conditions are trivially satisfied. □

Proof of Corollary 1. The proof consists of showing that the competitive equilibrium conditions with taxes and transfers specified by (36) are satisfied by the constrained-efficient allocation.

First, evaluate the intratemporal consumption decision (35) at the taxes in (36) to get

$$\frac{u^i_{dt}}{u^i_{ct}}=\frac{1+\gamma\tau_t}{1-(1-\gamma)\frac{\tau_t}{1+\tau_t}}, \tag{82}$$

which simplifies to

$$\frac{u^i_{dt}}{u^i_{ct}}=1+\tau_t. \tag{83}$$

which holds as it coincides with (95).

Second, to see that the budget constraint is satisfied, substitute the taxes and transfers in (36) into (34), to get

$$(1+\gamma\tau_t)c^i_{dt} + [1-(1-\gamma)\frac{\tau_t}{1+\tau_t}]c^i_{ct} = \varepsilon^i + \tau_{ct}\bar{c}, \tag{84}$$

which simplifies to

$$\begin{aligned} c^i_{dt} + c^i_{ct} - \varepsilon^i &= \frac{\tau_t}{1+\tau_t}\left\{(1-\gamma)(c^i_{ct}+\bar{c}) - \gamma c^i_{dt}(1+\tau_t)\right\} \\ &= 0. \end{aligned} \tag{85}$$

The last equality results from the preferences specified in (31), in which case, (35) implies

$$(1-\gamma)\left(c^i_{ct}+\bar{c}\right) = \gamma c^i_{dt}(1+\tau_t). \tag{86}$$

Market equilibrium conditions (5)-(7) hold as the government budget constraint is satisfied and (85) holds for all i. □

Proof of Proposition 3. The first part of the proof comes from comparing the intratemporal optimality condition on clean and dirty consumption in the quantitative model economy – which is the same as in the simple model, equation (8) – with the constrained optimal on (60). Both coincide if the carbon tax equals

$$\tau^i_t = \frac{\upsilon\sigma_t}{u^i_{ct}}$$

It follows that the constrained optimal carbon tax equals the one characterized in Proposition 1.

To show that income taxes take the form in the proposition, consider the equilibrium optimality conditions for labor and savings decisions:

$$\frac{v^i_{nt}}{u^i_{ct}} = (1-T'^n_t)F_{Nt}\varepsilon^i_t \tag{87}$$

$$u^i_{ct} \geq \beta\mathbf{E}_t\left\{u^i_{c,t+1}\left[(1-\tau^k_{t+1})F_{K,t+1}+1-\delta_k\right]\right\} \tag{88}$$

with the last equation holding with equality if the borrowing constraint is not binding. We consider a flat labor income tax, a capital income tax over total capital income (including depreciation), both of which may be individual-specific. We also assume that the borrowing constraints are not binding. These assumptions are without loss of generality to obtain simple tax formulas. Thus, the market optimality conditions can be written as

$$\frac{v^i_{nt}}{u^i_{ct}} = (1-\tau^i_{nt})F_{Nt}\varepsilon^i_t \tag{89}$$

$$u^i_{ct} = \beta \mathbf{E}_t \left\{ u^i_{c,t+1} \left[(1-\tau^i_{k,t+1})(F_{K,t+1}+1-\delta_k) \right] \right\} \tag{90}$$

Comparing the constrained-optimal Euler equation (59) with the market optimality condition (90), it follows that both coincide when the capital income tax/subsidy equals

$$\tau^i_{k,t+1} = \Lambda^i_{k,t+1} \tag{91}$$

Similarly, a labor income tax/subsidy is required to implement the constrained-optimal allocation. The optimal tax rate comes from comparing the constrained optimality condition (58) with (89) and equals

$$\tau^i_{nt} = \Lambda^i_{nt} \tag{92}$$

Tax revenues are rebated lump-sum to each household:

$$\tau^i_{nt}F_{Nt}\varepsilon^i_t n^i_t + \tau^i_{kt}(F_{Kt}+1-\delta)k^i_t + \tau^i_{dt}c^i_{dt} = T^i_t \tag{93}$$

so that the budget constraint of the government holds for every period t.

Given that the budget constraint holds for all i and the government budget constraint holds, market clearing conditions are trivially satisfied. This completes the proof. □

Proof of Corollary 2. To prove that the uniform carbon tax takes the form in (33), combine equations (65) and (66) to obtain:

$$\frac{u^i_{dt}}{u^i_{ct}} = 1 + \frac{1}{\alpha_i u^i_{ct}} \left[\upsilon\mu_i\sigma_t - \sum_{j\neq i} \eta^{ij}_t (c^j_{ct}+\bar{c}+c^j_{dt}) + \sum_{j\neq i} \eta^{ji}_t (c^j_{ct}+\bar{c}+c^j_{dt}) \right] \tag{94}$$

where the social cost of carbon, σ_t, is given by (61). Uniform taxation implies that the marginal rate of substitution between dirty and clean consumption must be equal for all i. Thus,

$$\frac{u^i_{dt}}{u^i_{ct}} = 1 + \tau_t \tag{95}$$

Following the proof of Proposition 2, plug this into equation (94) and multiplying by $\mu_i u^i_{ct}(c^i_{ct} + \bar{c} + c^i_{dt})$ and sum across all i to obtain:

$$\begin{aligned}\sum_i \tau_t \mu_i u^i_{ct}\left(c^i_{ct} + \bar{c} + c^i_{dt}\right) =& \upsilon\sigma_t \sum_i \mu_i \left(c^i_{ct} + \bar{c} + c^i_{dt}\right) \\ &- \sum_i \left(c^i_{ct} + \bar{c} + c^i_{dt}\right) \sum_{j\neq i} \eta^{ij}_t (c^j_{ct} + \bar{c} + c^j_{dt}) \\ &+ \sum_i \left(c^i_{ct} + \bar{c} + c^i_{dt}\right) \sum_{j\neq i} \eta^{ji}_t (c^j_{ct} + \bar{c} + c^j_{dt}) \\ =& \upsilon\sigma_t \sum_i \mu_i \left(c^i_{ct} + \bar{c} + c^i_{dt}\right)\end{aligned} \tag{96}$$

where the last equality comes from algebra manipulation. Reorganizing terms, we get

$$\tau_t \equiv \frac{\upsilon\sigma_t}{\sum_i \frac{\mu_i c^i_t}{\sum_j \mu_j c^j_t} u^i_{ct}} \tag{97}$$

where $c^i_t \equiv c^i_{ct} + \bar{c} + c^i_{dt}$. Thus, the uniform carbon tax takes the form of equation (33). □

D Sensitivity Analysis

In this section, we check the sensitivity of our main results—that a uniform carbon tax rebated back to households leads to a Pareto improvement—to alternative values of select parameters and to different model assumptions. First, we report how our results change for different coefficients of relative risk aversion. This parameter controls the degree to which the dispersion in consumption changes the uniform constrained-optimal carbon tax. Rows 2 and 3 of Table 6 show that lower (higher) risk aversion—all other parameters equal—lead to a higher (lower) carbon tax path. Both cases maintain full support for carbon taxation, and each delivers slightly higher average welfare gains compared to the baseline.

Next, we turn to the consequence of reducing the discount factor—all other parameters equal to the baseline. In this case, the optimal tax is lower than under the baseline because households (and the planner) discount future damages to the climate more aggressively. While this policy leads to smaller average welfare gains than in the baseline, it is still favored by all households.

Heterogeneous Climate Damages. Up to this point, we have assumed that climate damages are uniformly distributed over households. Here we relax this assumption by allowing climate damages to be type-dependent. Specifically, we suppose that the utility loss from carbon is

Table 6: Sensitivity Analysis

Policy	Initial carbon tax ($/ton)	Long-run temp. reduction (degrees)	Average welfare gain (percent)	Support (percent)
Baseline	41.0	1.0	0.045	100.0
Low risk aversion ($\kappa = 1.5$)	49.8	1.1	0.060	100.0
High risk aversion ($\kappa = 2.5$)	34.7	0.9	0.056	100.0
Low discount factor ($\beta = 0.96$)	32.5	0.8	0.029	100.0
Heterogeneous damages ($\zeta = 0.27$)	41.0	1.0	0.043	100.0
Heterogeneous damages ($\zeta = -0.15$)	41.0	1.0	0.047	100.0

$$x_i(S) = \frac{\Psi_i}{2} S^2, \tag{98}$$

where

$$\Psi_i = \Psi \frac{\varepsilon_i^{-\zeta}}{\sum_j \alpha_j \varepsilon_j^{-\zeta}}.$$

The parameter ζ controls the degree to which climate damages depend on productivity. If $\zeta < 0$, then for a given level of the carbon stock, the utility loss rises with income, while the loss is larger for the poor if ζ is positive. Setting $\zeta = 0$ corresponds to the baseline case. This functional form has the useful property that changes in ζ do not alter the equilibrium but only change how welfare is distributed across households.

To discipline the value of ζ, we use survey data on willingness-to-pay (WTP) measures from Kotchen et al. (2013). In this survey, households are asked how much they would be willing to pay each year to reduce carbon emissions by 17 percent. The survey also collects a range of demographic information, including income, age, education, and household size. To convert these WTP measures into consumption equivalents, we take the following steps. First, using the CEX, we regress household consumption on log income and other household characteristics such as age, education, and household size. Then, we use the regression coefficients to infer consumption expenditures for each household in the WTP survey. Finally, we calculate each household's consumption equivalent by dividing its WTP by its consumption.

We compute the semi-elasticity of consumption equivalent with respect to log income to be -0.01 and calibrate ζ so that the model matches this semi-elasticity. This requires $\zeta = 0.27$, meaning that a 50 percent reduction in income is associated with a 21 percent increase in climate damage. In the baseline case ($\zeta = 0$), the semi-elasticity is 0.01, meaning that higher income households gain more from climate mitigation, because they value consumption less. We also consider the case where $\zeta = -0.15$, under which the semi-elasticity is 0.03. In this case, the welfare gains are skewed even more to the rich as they also suffer larger climate damages.

The results from these specifications are reported in the fifth and sixth rows of Table 6. On net, the cases with heterogeneous climate damage lead to changes in average welfare similar to the case with homogeneous climate damage, and, importantly, the uniform carbon tax and rebate policies are Pareto improving in all cases. The distribution of welfare gains from both heterogeneous climate damage cases can be seen in Figure 7. As can be expected, the welfare gains are decreasing in income when climate damages are larger for the poor (panel a) and are increasing with income when climate damages are larger for the rich (panel b).

Figure 7: Welfare with heterogeneous climate damages

(a) $\zeta = 0.27$ (b) $\zeta = -0.15$